NEHRU COLLEGE OF ENGINEERING AND RESEARCH CENTRE
(NAAC Accredited)
(Approved by AICTE, Affiliated to APJ Abdul Kalam Technological University, Kerala)

DEPARTMENT OF MECHANICAL ENGINEERING

COURSE MATERIALS

ME304 DYNAMICS OF MACHINERY

VISION OF THE INSTITUTION

To mould true citizens who are millennium leaders and catalysts of change through excellence in education.

MISSION OF THE INSTITUTION

NCERC is committed to transform itself into a center of excellence in Learning and Research in Engineering and Frontier Technology and to impart quality education to mould technically competent citizens with moral integrity, social commitment and ethical values.

We intend to facilitate our students to assimilate the latest technological know-how and to imbibe discipline, culture and spiritually, and to mould them in to technological giants, dedicated research scientists and intellectual leaders of the country who can spread the beams of light and happiness among the poor and the underprivileged.

ABOUT DEPARTMENT

- Established in: 2002
- Course offered : B.Tech in Mechanical Engineering

- Approved by AICTE New Delhi and Accredited by NAAC
- Affiliated to the University of Dr. A P J Abdul Kalam Technological University.

DEPARTMENT VISION

Producing internationally competitive Mechanical Engineers with social responsibility & sustainable employability through viable strategies as well as competent exposure oriented quality education.

DEPARTMENT MISSION

1. Imparting high impact education by providing conductive teaching learning environment.
2. Fostering effective modes of continuous learning process with moral & ethical values.
3. Enhancing leadership qualities with social commitment, professional attitude, unity, team spirit & communication skill.
4. Introducing the present scenario in research & development through collaborative efforts blended with industry & institution.

PROGRAMME EDUCATIONAL OBJECTIVES

PEO1: Graduates shall have strong practical & technical exposures in the field of Mechanical Engineering & will contribute to the society through innovation & enterprise.

PEO2: Graduates will have the demonstrated ability to analyze, formulate & solve design engineering / thermal engineering / materials & manufacturing / design issues & real life problems.

PEO3: Graduates will be capable of pursuing Mechanical Engineering profession with good communication skills, leadership qualities, team spirit & communication skills.

PEO4: Graduates will sustain an appetite for continuous learning by pursuing higher education & research in the allied areas of technology.

PROGRAM OUTCOMES (POS)

Engineering Graduates will be able to:

1. **Engineering knowledge**: Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization to the solution of complex engineering problems.
2. **Problem analysis**: Identify, formulate, review research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences.
3. **Design/development of solutions**: Design solutions for complex engineering problems and design system components or processes that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, societal, and environmental considerations.

4. **Conduct investigations of complex problems**: Use research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.
5. **Modern tool usage**: Create, select, and apply appropriate techniques, resources, and modern engineering and IT tools including prediction and modeling to complex engineering activities with an understanding of the limitations.
6. **The engineer and society**: Apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal and cultural issues and the consequent responsibilities relevant to the professional engineering practice.
7. **Environment and sustainability**: Understand the impact of the professional engineering solutions in societal and environmental contexts, and demonstrate the knowledge of, and need for sustainable development.
8. **Ethics**: Apply ethical principles and commit to professional ethics and responsibilities and norms of the engineering practice.
9. **Individual and teamwork**: Function effectively as an individual, and as a member or leader in diverse teams, and in multidisciplinary settings.
10. **Communication**: Communicate effectively on complex engineering activities with the engineering community and with society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions.
11. **Project management and finance**: Demonstrate knowledge and understanding of the engineering and management principles and apply these to one's own work, as a member and leader in a team, to manage projects and in multidisciplinary environments.
12. **Life-long learning**: Recognize the need for, and have the preparation and ability to engage in independent and life-long learning in the broadest context of technological change.

PROGRAM SPECIFIC OUTCOMES (PSO)

PSO1: graduates able to apply principles of engineering, basic sciences & analytics including multi variant calculus & higher order partial differential equations..

PSO2: Graduates able to perform modeling, analyzing, designing & simulating physical systems, components & processes.

PSO3: Graduates able to work professionally on mechanical systems, thermal systems & production systems.

COURSE OUTCOMES

CO1	Analyze the static forces propagated from link to link of a four bar linkage and gear drive.
CO2	Analyze the dynamic forces propagated from link to link of a four bar linkage.
CO3	Analyze the concept and procedure for balancing of rotating and reciprocating masses, flywheel inertia and effects on energy fluctuations.
CO4	Analyze the effect of gyroscopic forces on stability of vehicles, ships and airplane.
CO5	Analyze vibration of single degree freedom systems with and without damping and principles of vibrations isolation.
CO6	Analyze the vibration due to whirling of shafts, vibration of multi-degree freedom systems and vibration measurement.

MAPPING OF COURSE OUTCOMES WITH PROGRAM OUTCOMES

	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12	PSO1	PSO2	PSO3
CO1	3	3	2	2								3	3	2	3
CO2	3	3	2	2								3	3	2	3
CO3	3	3	2	2								3	3	2	3
CO4	3	3	2	2								3	3	2	3
CO5	3	3	2	2								3	3	2	3
CO6	3	3	2	2								3	3	3	2

Note: H-Highly correlated=3, M-Medium correlated=2, L-Less correlated=1

SYLLABUS

Course code	Course Name	L-T-P-Credits	Year of Introduction
ME304	DYNAMICS OF MACHINERY	2-1-0-3	2016

Prerequisite: ME301 Mechanics of Machinery

Course Objectives:

- To impart knowledge on force analysis of machinery, balancing of rotating and reciprocating masses, Gyroscopes, Energy fluctuation in Machines.
- To introduce the fundamentals in vibration, vibration analysis of single degree of freedom systems.
- To understand the physical significance and design of vibration systems with desired conditions

Syllabus

Force analysis of machinery - static and dynamic force analysis of plane motion mechanisms. Flywheel analysis - static and dynamic balancing - balancing of rotating masses, gyroscopic couples. Vibrations – free vibrations of single degree freedom systems, damping, forced vibration, torsional vibration.

Expected outcome:

The students will be able to

1. Develop the design and practical problem solving skills in the area of mechanisms
2. Understand the basics of vibration and apply the concepts in design problems of mechanisms.

Text Books:

1. Ballaney P.L. Theory of Machines, Khanna Publishers,1994
2. S. S. Rattan, Theory of Machines, Tata McGraw Hill, 2009
3. V. P. Singh, Theory of Machines, Dhanpat Rai,2013

References :

1. E. Wilson, P. Sadler, Kinematics and Dynamics of Machinery, Pearson Education, 2003
2. Ghosh, A. K. Malik, Theory of Mechanisms and Machines, Affiliated East West Press, 2003
3. H. Myskza, Machines and Mechanisms Applied Kinematic Analysis, Pearson Education, 4e, 2012
4. Holowenko, Dynamics of Machinery, John Wiley, 1995
5. J. E. Shigley, J. J. Uicker, Theory of Machines and Mechanisms, McGraw Hill,1995
6. W.T.Thompson, Theory of vibration, Prentice Hall,1997

Course Plan			
Module	**Contents**	**Hours**	**End Sem. Exam Marks**
I	Introduction to force analysis in mechanisms - static force analysis (four bar linkages only) - graphical methods	4	15%
	Matrix methods - method of virtual work - analysis with sliding and pin friction	3	
II	Dynamic force analysis: Inertia force and inertia torque. D'Alemberts principle, analysis of mechanisms (four bar linkages only), equivalent dynamical systems	4	15%
	Force Analysis of spur- helical - bevel and worm gearing	3	
FIRST INTERNAL EXAM			
III	Flywheel analysis - balancing - static and dynamic balancing - balancing of masses rotating in several planes	4	15%
	Balancing of reciprocating masses - balancing of multi-cylinder in line engines - V engines - balancing of machines	3	
IV	Gyroscope – gyroscopic couples	3	15%
	Gyroscopic action on vehicles-two wheelers, four wheelers, air planes and ships. Stability of an automobile – stability of a two wheel vehicle –Stabilization of ship.	4	
SECOND INTERNAL EXAM			
V	Introduction to vibrations – free vibrations of single degree freedom systems – energy Method	2	20%
	Undamped and damped free vibrations – viscous damping – critical damping - logarithmic decrement - Coulomb damping – harmonically excited vibrations	3	
	Response of an undamped and damped system – beat phenomenon - transmissibility	2	
VI	Whirling of shafts – critical speed - free torsional vibrations – self excitation and stability analysis - vibration control - vibration isolation – vibration absorbers	4	20%
	Introduction to multi-degree freedom systems - vibration measurement - accelerometer – seismometer – vibration exciters	3	
END SEMESTER EXAM			

Question Paper Pattern

Maximum marks: 100 **Time: 3 hrs**

The question paper should consist of three parts

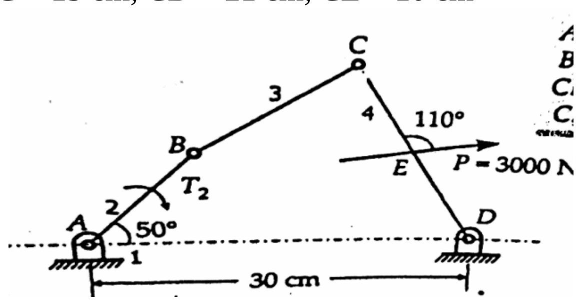
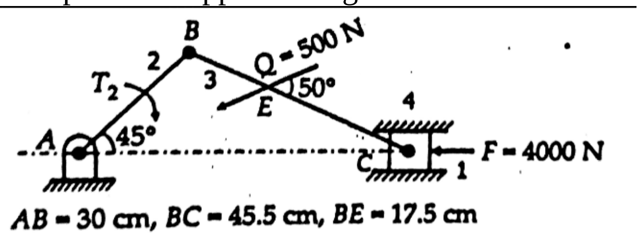
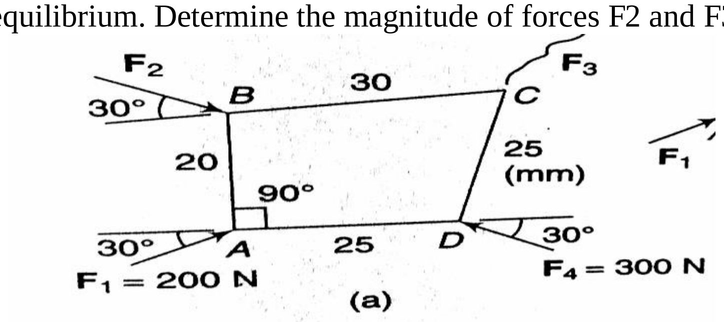
Part A
There should be 2 questions each from module I and II
Each question carries 10 marks
Students will have to answer any three questions out of 4 (3X10 marks =30 marks)

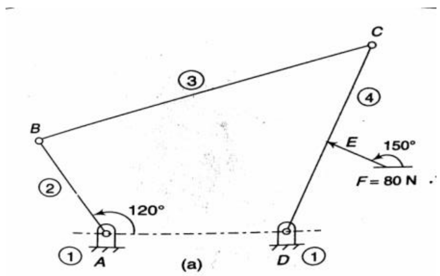
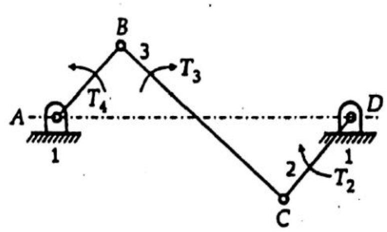
Part B
There should be 2 questions each from module III and IV
Each question carries 10 marks
Students will have to answer any three questions out of 4 (3X10 marks =30 marks)

Part C
There should be 3 questions each from module V and VI
Each question carries 10 marks
Students will have to answer any four questions out of 6 (4X10 marks =40 marks)

Note: Each question can have a maximum of four sub questions, if needed.

QUESTION BANK

MODULE I			
Q:NO:	**QUESTIONS**	**CO**	**KL**
1	What do you mean by static force analysis?	CO1	K2
2	What is principle of superposition?	CO1	K2
3	Explain in detail - principle of virtual work. Derive an expression to find out torque and force on a slider crank mechanism by principle of virtual work.	CO1	K6
4	What are free body diagrams of a mechanism? How are they helpful in finding the various forces on the various members of a mechanism?	CO1	K2
5	Discuss the equilibrium of a four force member?	CO1	K2
6	Determine T2 and various forces for the system shown in fig. And AB = 12 cm, BC = 15 cm, CD = 21 cm, CE = 10 cm 	CO1	K5
7	Determine the couple T2 as applied in fig 	CO1	K5
8	Fig shows a quaternary link ABCD under the action of forces F1,F2,F3 and F4 acting at A,B,C and D respectively. The link is in static equilibrium. Determine the magnitude of forces F2 and F3. 	CO1	K5

9	A four link mechanism as shown in fig. It is acted up on by a force of 80N<150 deg on link DC. AD = 50 mm, AB =40 mm, BC = 100 mm, DC= 75 mm, DE = 35 mm. Determine the input torque T on the link AB for static equilibrium of mechanism for the given configure 	CO1	K5
10	In the four bar mechanism shown in fig. torque T3 & T4 are 50 N-m and 60 N-m respectively. For the static equilibrium of system, find the required input torque T2. BC= 80 cm, AB=35cm, AD=109 cm, CD= 38cm $BC = 80$ cm , $AB = 35$ cm , $AD = 109$ cm, $CD = 38$ cm 	CO1	K5
MODULE II			
1	State and Explain D'Alemberts principle.	CO2	K3
2	Derive an expression to find out crank effort in an IC engine	CO2	K6
3	What is dynamic force analysis?	CO2	K2
4	What is inertia? Explain about inertia force.	CO2	K3
5	What do you mean by dynamically equivalent system? What do you mean by equivalent offset inertia force?	CO2	K2
6	A horizontal gas engine running at 210 rpm has a bore of 220 mm and stroke of 440 mm. the connecting rod is 924 mm long and reciprocating parts weigh 20 kg. When the crank has turned through an angle of 30 deg from inner dead center, the gas pressures on the cover and crank sides are 500 KN/ m^2 and 60 KN/m^2 respectively.	CO2	K5

	Diameter of piston rod is 40 mm. determine a. turning moment on crank shaft b. thrust on bearings c. acceleration of flywheel which has a mass of 8 kg and radius of gyration of 600 mm while the power of gas engine is 22 KW		
7	The crank and connecting rod of a vertical petrol engine running at 1800 rpm are 60 mm and 270 mm respectively. The diameter of piston is 100 mm and the mass of reciprocating parts is 1.2 kg. During the expansion stroke when the crank has turned through 20 deg from top dead center, the gas pressure is 650 kN/m^2. Determine a. the net force on piston b. the net load on gudgeon pin c. the thrust on cylinder walls d. the speed at which gudgeon pin load is reversed in direction	CO2	K5
8	How do we perform force analysis on a spur gear? Derive the expressions to find out tangential force, radial force and resultant forces on a spur gear	CO2	K6
9	How do we perform force analysis on a helical gear? Derive the expressions to find out tangential force, radial force , axial force and resultant forces on a helical gear	CO2	K6
10	Derive an expression to find out torque and force components in a reciprocating steam engine without considering the weight of connecting rod	CO2	K6
	MODULE III		
1	Explain the turning moment diagram of a four stroke cycle IC engines.	CO3	K3
2	What is the function of a flywheel and how does it differ from that of a governor?	CO3	K2
3	Prove that maximum fluctuation of energy $\Delta E = E * 2 Cs$	CO3	K2
4	Derive the following expressions, for an uncoupled two cylinder locomotive engine a. Variation in tractive force b. Swaying couple c. Hammer blow	CO3	K6
5	Discuss the balancing of V engines	CO3	K2
6	A shaft carries 4 masses A,B,C and D of magnitude 200kg,300 kg,400 kg and 200 kg respectively and revolving at radii 80 mm, 70 mm, 60 mm, 80 mm in planes measured from A at 300 mm, 400 mm, 700 mm. The angles between cranks measured anticlockwise are A to B 45 deg, B to C 70 deg, C to D is 120 deg. The balancing masses are to be placed in planes X and Y. The distance between planes A and X is 100 mm, between X and Y 400 mm and between Y and D is 200 mm. If the balancing masses revolve at a radius of 100 mm, find their magnitudes and angular positions	CO3	K5
7	The cranks and connecting rods of a 4 cylinder in line engine running at 1800 r.p.m are 60 mm and 240 mm each respectively and	CO3	K5

	the cylinders are spaced 150 mm apart. If the cylinders are numbered 1 to 4 in sequence from one end, the cranks appear at intervals of 90 deg in an end view in the order 1-4-2-3. The reciprocating mass corresponding to each cylinder is 1.5 kg. Determine a. Unbalanced primary and secondary forces b. Unbalanced Primary and secondary couples with reference to central plane of engine		
8	The turning moment diagram for a four stroke engine may be assumed for simplicity to be represented by four triangles, the areas of which from the line of zero pressure are as follows: Suction stroke = 0.45* 10^ -3 m2, compression stroke = 1.7 * 10 ^-3 m2, Expansion stroke = 6.8 * 10^-3 m2 and exhaust stroke = 0.65* 10 ^-3 m2. Each m2 of area represents 3 MN-m of energy. Assuming the resisting torque to be uniform, find the mass of rim of flywheel required to keep the speed between 202 and 198 rpm. The mean radius of rim is 1.2 m.	CO3	K5
9	A shaft carries 4 masses in parallel planes A, B, C and D in this order along its length. The masses at B and C are 18 kg and 12.5 kg respectively and each has an eccentricity of 80 mm. The angle between the masses at B and C is 100 deg and that between the masses at B and A is 190 deg, both being measured in same direction. The axial distance between the planes A and B is 100 mm and that between B and C is 200 mm. If the shaft is in complete dynamic balance, determine a. The magnitude of masses at A and D b. The distance between planes A and D. The angular position of mass at D.	CO3	K5
10	A,B,C and D are four masses carried by a rotating shaft at radii 100, 125, 200 and 150 mm respectively. The planes in which the masses revolve are spaced 600 mm apart and the mass of B, C and D are 10 kg, 5 kg and 4 kg respectively. Find the required mass A and the relative angular settings of the four masses so that the shaft shall be in complete balance	CO3	K5
	MODULE IV		
1	What do you understand by gyroscopic couple? Derive a formula for its magnitude.	CO4	K6
2	Describe the application of gyroscopic principles to aircrafts.	CO4	K2
3	Describe the gyroscopic effect on sea going vessels.	CO4	K2
4	Discuss the effect of the gyroscopic couple on a two wheeled vehicle when taking a turn.	CO4	K2
5	What will be the effect of the gyroscopic couple on a disc fixed at a certain angle to a rotating shaft?	CO4	K1
6	A ship propelled by a turbine rotor which has a mass of 5 tonnes	CO4	K5

	and a speed of 2100 r.p.m. The rotor has a radius of gyration of 0.5 m and rotates in a clockwise direction when viewed from the stern. Find the gyroscopic effects in the following conditions: 1. The ship sails at a speed of 30 km/h and steers to the left in a curve having 60 m radius. 2. The ship pitches 6 degree above and 6 degree below the horizontal position. The bow is descending with its maximum velocity. The motion due to pitching is simple harmonic and the periodic time is 20 seconds. 3. The ship rolls and at a certain instant it has an angular velocity of 0.03 rad/s clockwise when viewed from stern. Determine also the maximum angular acceleration during pitching. Explain how the direction of motion due to gyroscopic effect is determined in each case.		
7	Explain the effect of the gyroscopic couple on the reaction of the four wheels of a vehicle negotiating a curve	CO4	K3
8	An aeroplane makes a complete half circle of 50 metres radius, towards left, when flying at 200 km per hour. The rotary engine and the propeller of the plane have a mass of 400 kg with a radius of gyration of 300 mm. The engine runs at 2400 r.p.m. clockwise, when viewed from the rear. Find the gyroscopic couple on the aircraft and state its effect on it. What will be the effect, if the aeroplane turns to its right instead of to the left?	CO4	K5
9	The mass of the turbine rotor of a ship is 20 tonnes and has a radius of gyration of 0.60 m. Its speed is 2000 r.p.m. The ship pitches 6° above and 6° below the horizontal position. A complete oscillation takes 30 seconds and the motion is simple harmonic. Determine the following: 1. Maximum gyroscopic couple, 2. Maximum angular acceleration of the ship during pitching, and 3. The direction in which the bow will tend to turn when rising, if the rotation of the rotor is clockwise when looking from the left.	CO4	K5
10	The turbine rotor of a ship has a mass of 3500 kg. It has a radius of gyration of 0.45 m and a speed of 3000 r.p.m. clockwise when looking from stern. Determine the gyroscopic couple and its effect upon the ship: 1. When the ship is steering to the left on a curve of 100 m radius at a speed of 36 km/h. 2. When the ship is pitching in a simple harmonic motion, the bow falling with its maximum velocity. The period of pitching is 40 seconds and the total angular displacement between the two extreme positions of pitching is 12 degrees	CO4	K5
	MODULE V		
1	Deduce an expression for the natural frequency of free transverse vibrations for a simply supported shaft carrying uniformly distributed mass of m kg per unit length.	CO5	K6

2	Explain the term 'Logarithmic decrement' as applied to damped vibrations	CO5	K3
3	Explain the terms 'under damping, critical damping' and 'over damping'	CO5	K3
4	Derive the differential equation characterizing the motion of an oscillation system subject to viscous damping and no periodic external force. Assuming the solution to the equation, find the frequency of oscillation of the system.	CO5	K6
5	Discuss briefly with neat sketches the longitudinal, transverse and torsional free vibrations	CO5	K2
6	A vertical shaft of 5 mm diameter is 200 mm long and is supported in long bearings at its ends. A disc of mass 50 kg is attached to the centre of the shaft. Neglecting any increase in stiffness due to the attachment of the disc to the shaft, find the critical speed of rotation and the maximum bending stress when the shaft is rotating at 75% of the critical speed. The centre of the disc is 0.25 mm from the geometric axis of the shaft. E = 200 GN/ m2.	CO5	K5
7	Explain the term 'whirling speed' or 'critical speed' of a shaft. Prove that the whirling speed for a rotating shaft is the same as the frequency of natural transverse vibration.	CO5	K3
8	The mass of an electric motor is 120 kg and it runs at 1500 r.p.m. The armature mass is 35 kg and its C.G. lies 0.5 mm from the axis of rotation. The motor is mounted on five springs of negligible damping so that the force transmitted is one-eleventh of the impressed force. Assume that the mass of the motor is equally distributed among the five springs. Determine: 1. stiffness of each spring; 2. dynamic force transmitted to the base at the operating speed; and 3. natural frequency of the system.	CO5	K5
9	The mass of a single degree damped vibrating system is 7.5 kg and makes24 free oscillations in 14 seconds when disturbed from its equilibrium position. The amplitude of vibration reduces to 0.25 of its initial value after five oscillations. Determine: 1. stiffness of the spring, 2. logarithmic decrement, and 3. damping factor, i.e. the ratio of the system damping to critical damping.	CO5	K5
10	A shaft 50 mm diameter and 3 metres long is simply supported at the ends and carries three loads of 1000 N, 1500 N and 750 N at 1 m, 2 m and 2.5 m from the left support.The Young's modulus for shaft material is 200 GN/m2. Find the frequency of transverse vibration.	CO5	K5
MODULE VI			
1	What is meant by torsionally equivalent length of a shaft as referred to a stepped shaft? Derive the expression for the equivalent length of a shaft which has several steps.	CO6	K6

2	How the natural frequency of torsional vibrations for a two rotor system is obtained?	CO6	K2
3	Derive an expression for the natural frequency of free transverse and longitudinal vibrations by equilibrium method	CO6	K6
4	Derive an expression for the frequency of free torsional vibrations for a shaft fixed at one end and carrying a load on the other free end	CO6	K6
5	Discuss the effect of inertia of a shaft on the free torsional vibrations.	CO6	K2
6	A steel shaft of 1.5m long is 95 mm in diameter for the first 0.6m of its length, 60 mm in diameter for the next 0.5 m of the length and 50 mm in diameter for the remaining 0.4m of its length. The shaft carries two fly wheels at two ends, the first having a mass of 900kg and 0.85 m radius of gyration located at the 95 mm diameter end and the second having a mass of 700 kg and 0.55m radius of gyration located at the other end. Determine the location of the node and natural frequency of free torsional vibration of the system. The modulus of rigidity of shaft material may be taken as 80 GN/m2	CO6	K5
7	Establish the expression to determine the frequency of torsional vibrations of a geared system	CO6	K6
8	Describe the method of finding the natural frequency of torsional vibrations for a three rotor system.	CO6	K2
9	Three rotors A, B and C having moment of inertia of 2000, 6000 and 3500 kg-m2 respectively are carried on a uniform shaft of 0.35 m diameter. The length of shaft between rotor A and B is 6 m and between B and C is 32m. Find the natural frequency of torsional vibrations. Modulus of rigidity of shaft material is 80 GN/m2	CO6	K5
10	Three rotors A, B and C having moment of inertia of 2500, 6700 and 3800 kg-m2 respectively are carried on a uniform shaft of 0.35 m diameter. The length of shaft between rotor A and B is 6 m and between B and C is 35m. Find the natural frequency of torsional vibrations. Modulus of rigidity of shaft material is 85 GN/m2	CO6	K5

Module 1

Introduction to force analysis in mechanisms - static force analysis (four bar linkages only) - graphical methods

Matrix methods - method of virtual work - analysis with sliding and pin friction

Module. 1

Static Force Analysis

In all type of machinary, forces are transmitted from one component to another.

eg: Belt to pulley, brakedrum - brake shoe, Gear to shaft etc.

In the design of mechanism it is necessary to know magnitude as well as direction of transmitted force from i/p to o/p. This analysis helps ~~to~~ in selection of size, shape and type of material of the machine component to withstand the forces and stresses.

If the component of a machine accelerate inertia ~~froms~~ forces are produced due to masses. If the magnitude of these forces are small compared to external applied load - the inertia force can be neglected while analysis. Such analysis is known as static force analysis.

When inertia forces are also considered along with external forces is called dynamic force analysis

Static force analysis

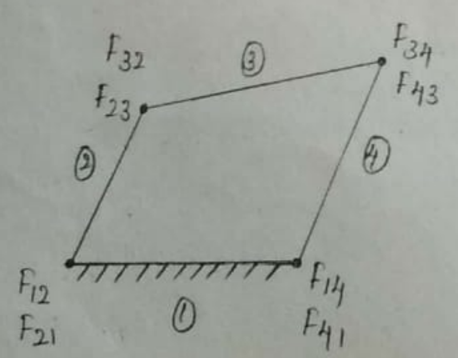
1. Constraint forces

These are the forces considering at the joints of machine components.

2. Applied forces.

These are the forces applied in external manner. which means by electrically, mechanically, and magnetically.

Consider a 4 bar mechanism

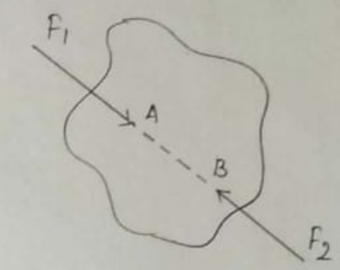


$f_{12}, F_{21}, F_{32}, F_{23} \cdots$ are Constraint forces

Conditions for static equilibrium

1. Sum of forces $\sum F = 0$
2. Sum of all moment $\sum M = 0$.

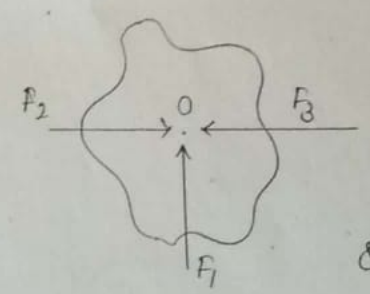
Equilibrium of 2 force member.



A member under the action of 2 forces will be in equilibrium.

If forces have equal magnitude ($F_1 = F_2$) and are inline of action, but opposite in direction.

Equilibrium of 3 force member.



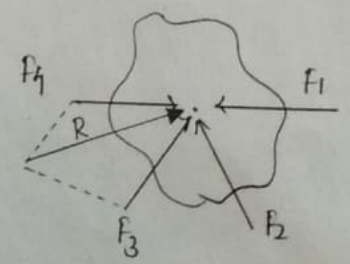
A member under the action of 3 forces will be in equilibrium.

If sum of the forces should be zero;

$F_1 + F_2 + F_3 = 0$.

All these forces should meet at a ~~poin~~ common point O- point of concurrency.

Equilibrium of 4 force member

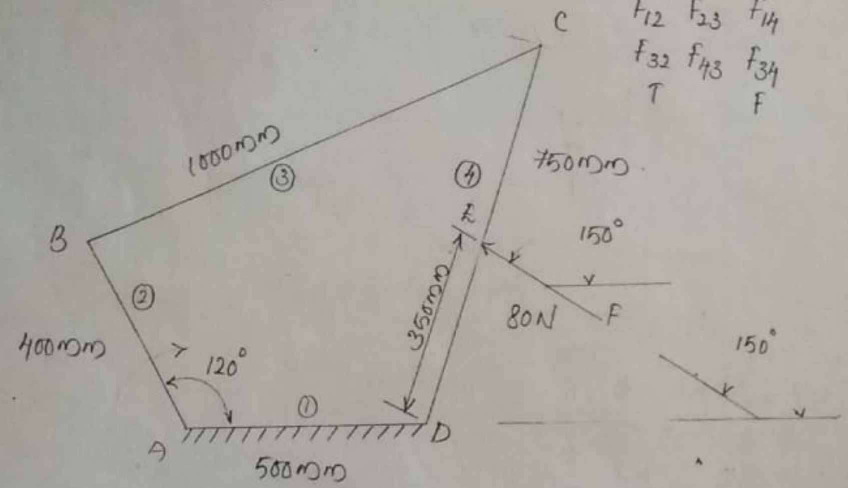


Reduced into 3 force member.

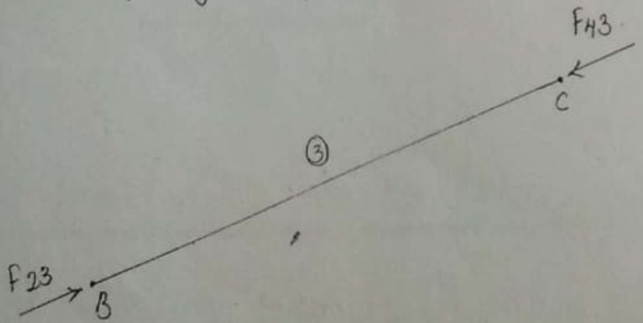
* A 4 bar mechanism with the following dimensions is acted upon by a force 80N∠150° on the link DC, AD = 500mm, AB = 400mm, BC = 1000mm, DC = 750mm and DE = 350mm. Determine the i/p torque T on the link AB for static equilibrium.

scale 1000mm = 1cm

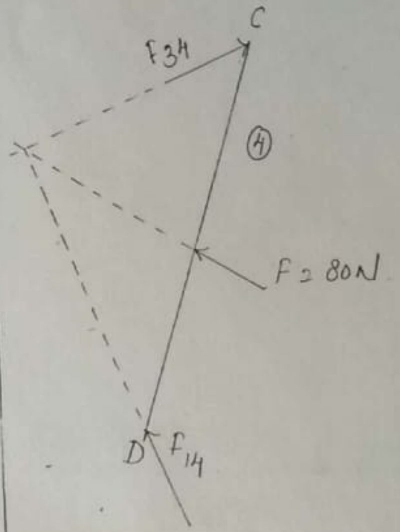
②	③	④
F_{12}	F_{23}	F_{14}
F_{32}	F_{43}	F_{34}
T		F



Free body diagram of link ③



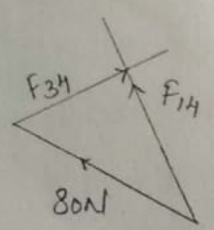
Free body diagram of link ④



force polygon

Scale

20N = 1cm



$F_{34} = 2.4cm$

$= 48N$

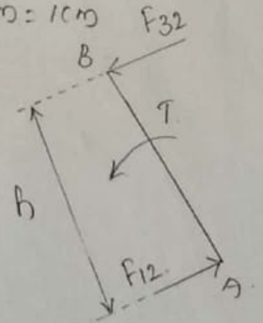
$F_{14} = 3.1cm$

$= 62N$

Free body diagram of link ②

Scale

100mm = 1cm



$F_{32} = F_{23} = F_{43} = F_{34}$

$F_{32} = 48N \quad h = 4cm$

$T = F_{32} \times h$

$= 48 \times 400$

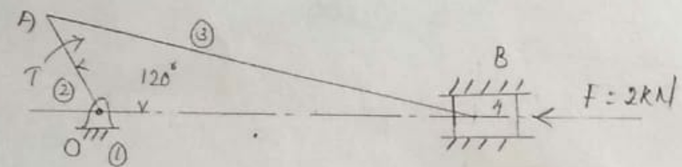
$= 19200 \ Nmm$

$= -19.2 \ Nm$

[anticlockwise]

Slider crank mechanism with the following dimensions is acted upon by a force $F = 2\,kN$ at B as shown in fig. $OA = 150\,mm$, $AB = 450\,mm$, determine the i/p torque T on the link OA.

Scale
50 mm = 1 cm



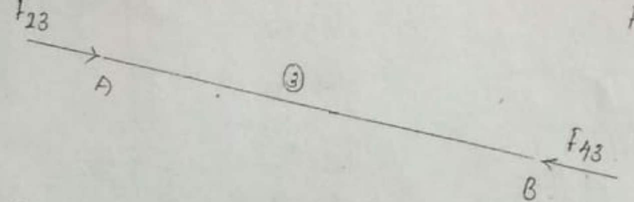
② ③ ④

F_{12} F_{43} F_{34}

F_{32} F_{23} F_{14}

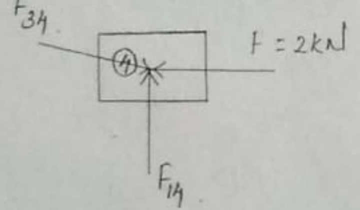
T F

FBD ③

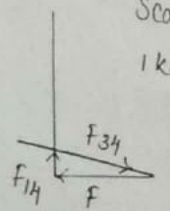


$F_{23} = F_{43}$

FBD ④



Scale
1 kN = 1 cm

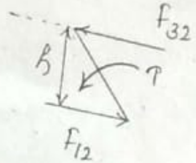


$F_{14} = 0.5\,cm$
$= .5\,kN$

$F_{34} = 2.1\,cm$
$= 2.1\,kN$

FBD ②

Scale
50 mm = 1 cm



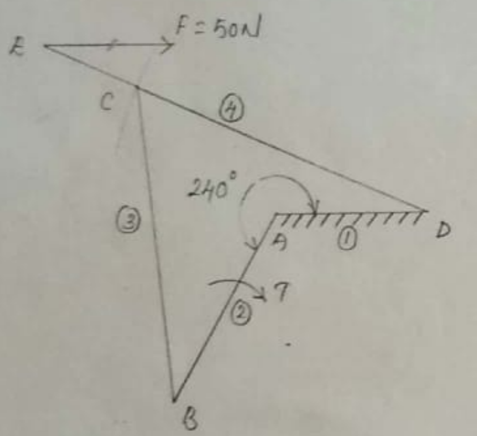
$h = 1.5\,cm$
$= 75\,mm$

$F_{32} = F_{23} = F_{43} = F_{34} = 2.1\,kN$

$T = F_{32} \times h$ [force × ⊥r dist]
$= 2.1 \times 75$
$= 157.5\,kN\,mm$

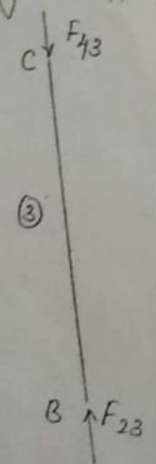
* 4 A link mechanism with the following dimensions is acted upon by a force of 50 N on the link DC at the Point E as shown in fig. AD = 300mm, AB = 400mm, BC = 600mm, DC = 640 mm, DE = 840 mm. Determine the i/p torque T on the link AB.

Scale 100mm = 1cm

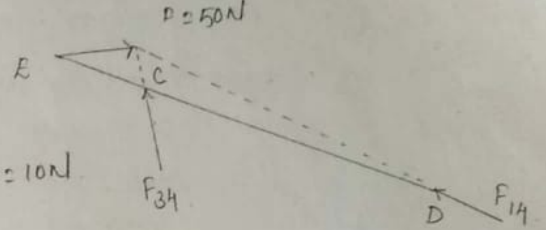


②	③	④
F_{12}	F_{43}	F_{14}
F_{32}	F_{23}	F_{34}
T		F

Free body diagram of link ③



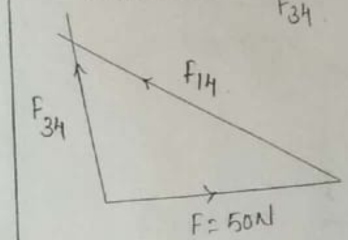
Free body diagram of link. ④



$F_{34} = 32$ N

$F_{14} = 62$ N

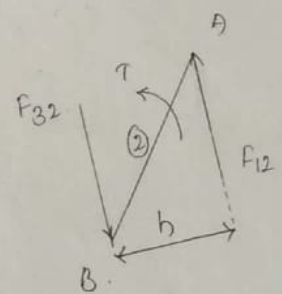
Scale 1cm = 10N



$F_{34} = 3.2$ cm

$F_{14} = 6.2$ cm

Free body diagram of link ②



$h = 2.45$ cm

$= 245$ mm

$T = F_{32} \times h$

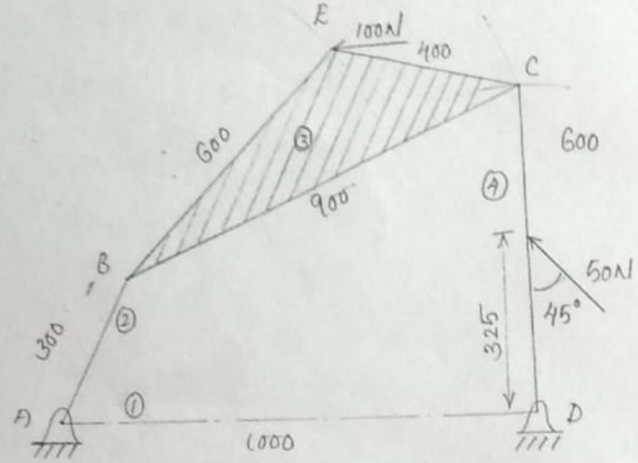
$= 32 \times 245$

$= 7840$ Nmm

$F_{32} = F_{23} = F_{43} = F_{34} = 32$ N

...d the torque on the link AB for the fig. shown

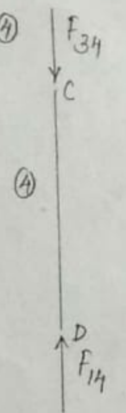
Scale 1cm = 100mm.



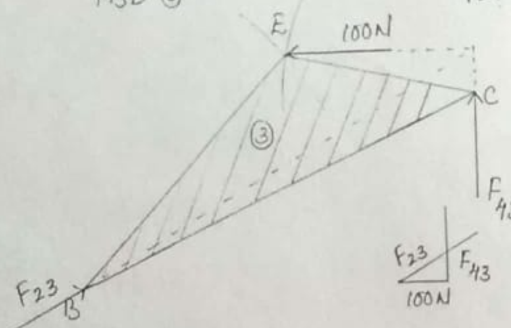
②	③	④
F_{12}	F_{23}	F_{14}
F_{32}	F_{43}	F_{34}
T=?	100N	50N

Case.1 [Considering force 100N]

FBD ④



FBD ③



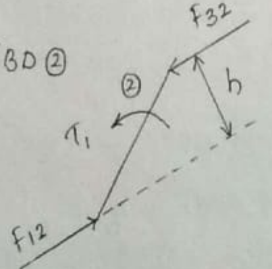
Scale
100N = 1cm

$F_{43} = 50N$

$F_{23} = 120N$

$F_{23} = F_{32} = 120N$

FBD ②



h = 1.6 cm

= 160mm

$T_1 = F_{32} \times h$

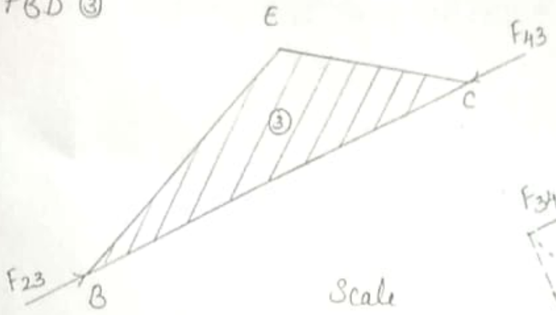
= 120 × 160

= −19200 N/mm

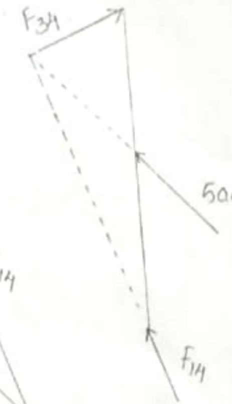
[anticlockwise]

Case.2 [Considering 50N only]

FBD ③

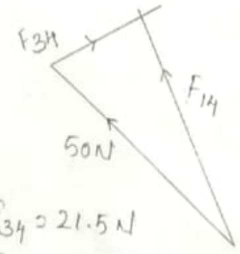


FBD ④



Scale

10N = 1cm



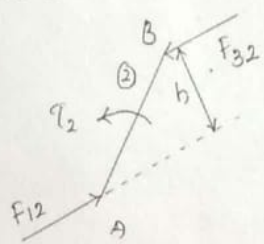
$F_{34} = 2.15$ cm

$F_{14} = 4.6$ cm

ie; $F_{34} = 21.5$ N

$F_{14} = 46$ N

FBD ②



h = 1.7cm

= 170 mm

$F_{32} = F_{23} = F_{43} = F_{34} = 21.5$ N

∴ $T_2 = F_{32} \times h$

= 21.5 × 170

= −3655 N/mm

$T = T_1 + T_2$

= −19200 − 3655

= −22855 N/mm

= − 22.855 Nm.

[anticlockwise]

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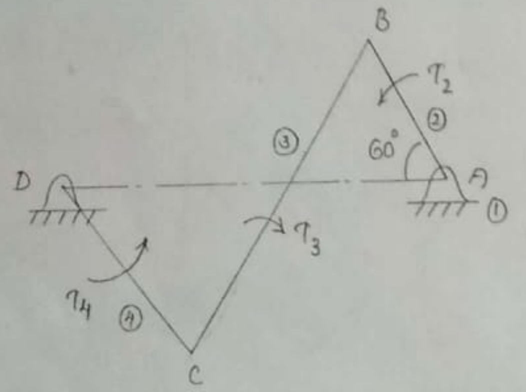
* $AD = 800\,mm$, $AB = 300\,mm$, $BC = 700\,mm$, $CD = 400\,mm$, $T_2 = ?$ $T_4 = 20\,N/m$ and $T_3 = 30\,N/m$.

Scale

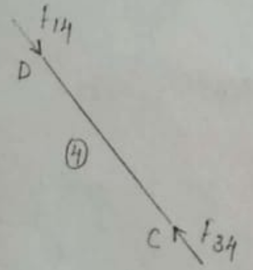
$1\,cm = 100\,mm$



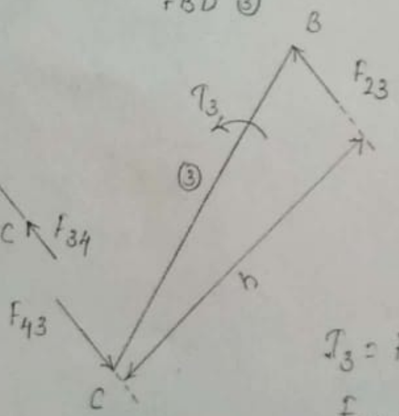
②	③	④
F_{32}	F_{43}	F_{14}
F_{12}	F_{23}	F_{34}
T_2	T_3	T_4

Case 1 [Consider torque T_3]

FBD ④



FBD ③

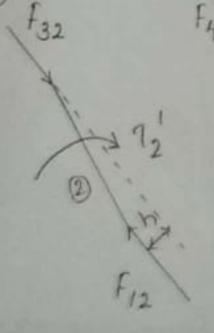


$h = 6.70\,cm$
$= 670\,mm$
$= 0.670\,m$

$T_3 = F_{23} \times h$

$F_{23} = T_3/h$
$= 30/0.67$
$= 44.7761\,N$

FBD ②



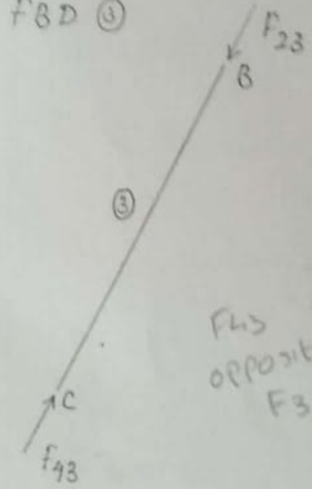
$h = 0.5\,cm$
$= 50\,mm$

$T_2' = F_{12} \times h$
$= 44.7761 \times 50$
$= 2238.805\,N/mm$

$T_2' = +2.238\,N/m$

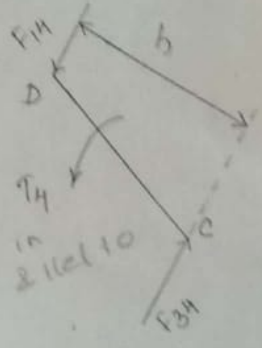
Case 2 [Consider torque T_4]

FBD ③



F_{43} is in opposite & llel to F_{34}

FBD ④

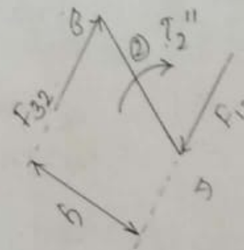


$h = 390\,mm$

$T_4 = F_{34} \times h$

$F_{34} = \frac{T_4}{h}$
$= \frac{20}{0.39}$
$= 51.2820\,N$

FBD ②



$h = 2.65\,cm$
$= 0.265\,m$

$F_{12} = F_{32} = F_{23} = F_{43}$
$= F_{34} = 51.2820\,N$

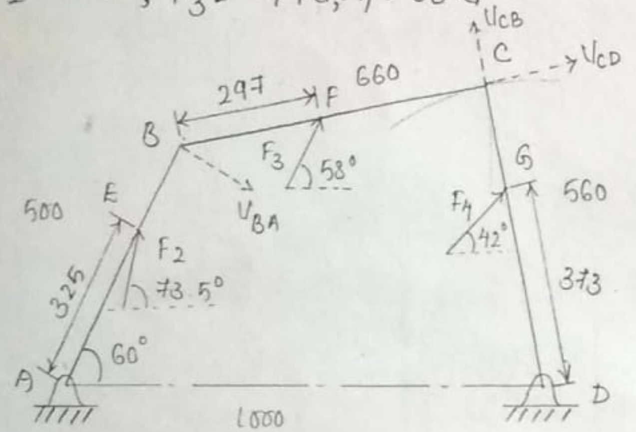
$T_2'' = F_{12} \times h$
$= 51.2820 \times 0.265$
$= +13.5897\,N/m$

$T_2 = T_2' + T_2''$
$= 2.238 + 13.5897$
$= 15.8277\,N/m$
Clockwise

Principle of Virtual Work

AB = 500 BC = 660 CD = 560 AD = 1000 [all dimensions are in mm] find torque of i/p link

$F_2 = 80N$, $F_3 = 144N$, $F_4 = 60N$



Scale

100mm = 1cm

Assume ω = 1 rad/s

$V = r\omega$

AB

$V_{BA} = BA \times \omega_{BA}$

$= 0.5 \times 1$

$= 0.5$ m/s

$\frac{AB}{AE} = \frac{ab}{ae}$ $\quad \frac{BC}{BF} = \frac{bc}{bf}$

$\frac{5}{3.25} = \frac{10}{ae}$ $\quad \frac{6.6}{2.97} = \frac{6.65}{bf}$

$ae = 6.5$ cm $\quad bf = 2.99$ cm

$\frac{DC}{DG} = \frac{dc}{dg}$

$\frac{5.6}{3.73} = \frac{7.75}{dg}$

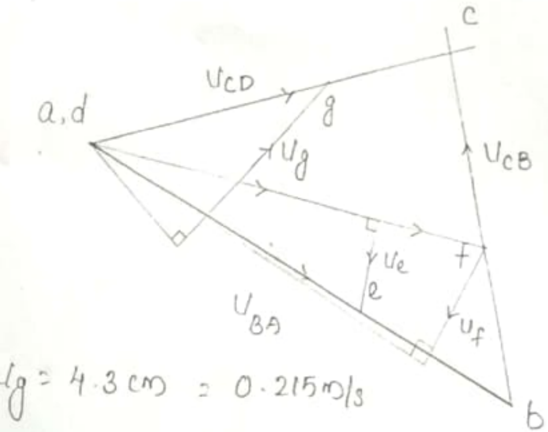
$dg = 5.16$ cm

Velocity diagram

Scale 20cm = 1m/s

$V_{CD} = 7.75$ cm

$V_{CB} = 6.65$ cm



$V_g = 4.3$ cm $= 0.215$ m/s

$V_e = 1.7$ cm $= 0.085$ m/s

$V_f = 2.5$ cm $= 0.125$ m/s

$$(T \times \omega) - (F_2 \times V_e) + (F_3 \times V_f) + (F_4 \times V_g) = 0$$

$$(T \times 1) - (80 \times 0.085) + (144 \times 0.125) + (60 \times 0.215) = 0$$

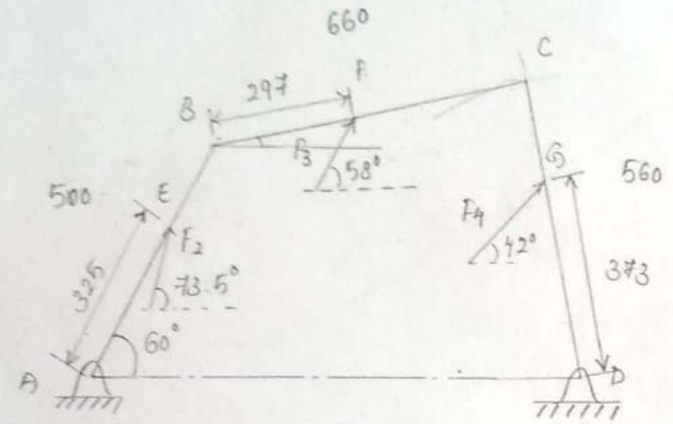
$\therefore T = -24.1$ Nm

$= 24$ Nm [anticlockwise]

Steps

1. Velocity diagram
2. Mark the points by relation ratio
3. Parallel lines at points and $\perp^r$ from fixed pt
4. Measure velocities at that pt [Parallel line]
5. Apply principle of virtual work

Matrix Method



Force vectors

$F_2 = 80 \angle 73.5^\circ = (80\cos 73.5)i + (80\sin 73.5)j$
$= 22.72\,i + 76.71\,j$

$F_3 = 144 \angle 58^\circ = 76.30\,i + 122.11\,j$

$F_4 = 60 \angle 42^\circ = 44.58\,i + 40.14\,j$

Position vectors

$AB = 0.5 \angle 60^\circ = 0.25\,i + 0.43\,j$

$AE = 0.325 \angle 60^\circ = 0.16\,i + 0.28\,j$

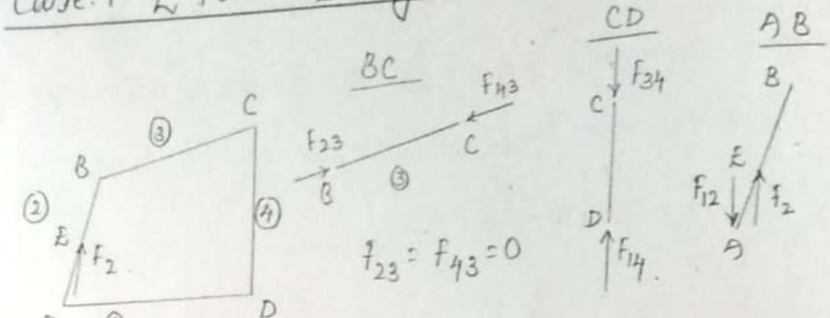
$BC = 0.66 \angle 11^\circ = 0.64\,i + 0.12\,j$

$BF = 0.297 \angle 11^\circ = 0.29\,i + 0.05\,j$

$DC = 0.56 \angle 100^\circ = -0.09\,i + 0.55\,j$

$DG = 0.373 \angle 100^\circ = -0.06\,i + 0.36\,j$

Case. 1 [force F_2 only]



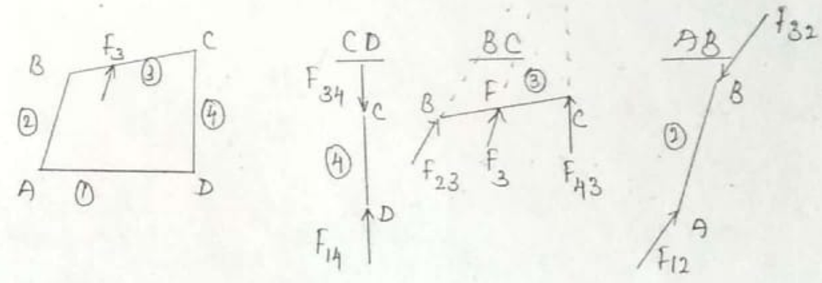
$F_{23} = F_{43} = 0$

$F_{34} = F_{14} = 0$

Torque $T_1 = F_2 \times AE$

$$= \begin{vmatrix} i & j & k \\ 22.72 & 76.71 & 0 \\ .16 & .28 & 0 \end{vmatrix} = -5.912\ \text{Nm}.$$

Case. 2 [force F_3 only]



$F_{34} = F_{14}$

$T_2 = F_{32} \times AB = -F_{23} \times AB$.

at BC
Moment at $B = 0$

$F_3 \times BF + F_{43} \times BC = 0$

$F_{43} = x \angle 100^\circ$

$= -x(0.1736)i + x(0.9848)j$

Sub: in $\sum M = 0$.

$76.30i + 122.11j$ ~~[illegible]~~ $+ (-x(0.1736)i) + x(0.9848)j) \times$

$0.29i + 0.05j \qquad 0.64i + 0.12j \qquad = 0$

$$\begin{vmatrix} i & j \\ 76.30 & 122.11 \\ 0.29 & 0.05 \end{vmatrix} + \begin{vmatrix} i & j \\ -x\,0.1736 & x\,0.9848 \\ 0.64 & 0.12 \end{vmatrix} = 0$$

$-31.5969 + (-0.021x) - (0.631x) = 0.$

$-31.5969 + -0.652x = 0.$

$0.652x = 31.5969$

$x = -48.4615\ N.$

$\sum F = 0$

$F_{23} + F_3 + F_{43} = 0.$

$\therefore F_{23} = -F_3 - F_{43}$

$= -(76.30i + 122.11j) - (+8.41i - 47.72j).$

$= -84.71i - 74.39j$

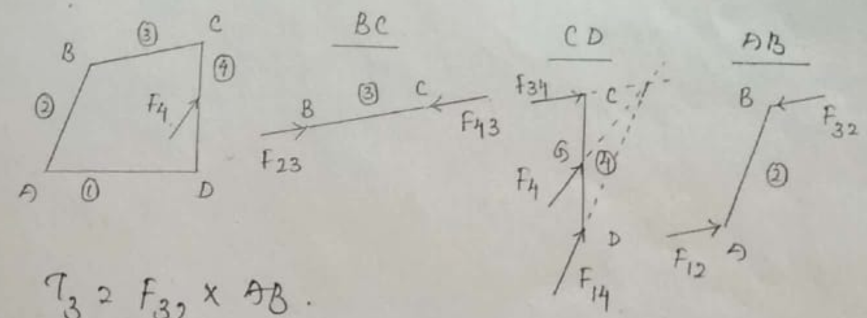
$T_2 = -F_{23} \times AB$

$= (-84.71i - 74.39j) \times (0.25i + 0.43j)$

$$= \begin{vmatrix} i & j \\ -84.71 & -74.39 \\ 0.25 & 0.43 \end{vmatrix}$$

$= +17.8278\ Nm.$

Case. 3 [force f_4 only].



$T_3 = F_{32} \times AB.$

$F_{32} = -F_{23} = F_{43} = -F_{34}$

Moment at D $= 0$.

$F_4 \times DG + F_{34} \times CD = 0.$

Net force on link 4

$F_{34} = x \angle 11^\circ.$

$= x\cos 11 i + x\sin 11 j$

$= 0.98xi + 0.19xj$

Sub : in $\sum M = 0$

$$\underbrace{\begin{vmatrix} i & j \\ 44.58 & 40.14 \\ -0.06 & 0.36 \end{vmatrix}}_{F_4 \times DG} + \underbrace{\begin{vmatrix} i & j \\ 0.98x & .19x \\ -0.09 & 0.55 \end{vmatrix}}_{F_{34} \times CD} = 0$$

$$18.4572 + (0.539x + 0.0171x) = 0$$

$$18.4572 + (0.5561x) = 0$$

$$0.5561x = -18.\cancel{4572}\ 4572$$

$$x = -33.19 \text{ N}.$$

$$F_{34} = -32.52i - 6.3061j$$

$$T_3 = -F_{34} \times AB$$

$$= -(-32.52i - 6.30j) \times (0.25i + 0.43j)$$

$$= \begin{vmatrix} i & j \\ 32.52 & 6.30 \\ 0.25 & 0.43 \end{vmatrix}$$

$$= 12.4086 \text{ Nm}.$$

Net torque $T = T_1 + T_2 + T_3$

$$= -5.912 + 17.8278 + 12.4086$$

$$= 24.3244 \text{ Nm}.$$

Anticlockwise

Module 2

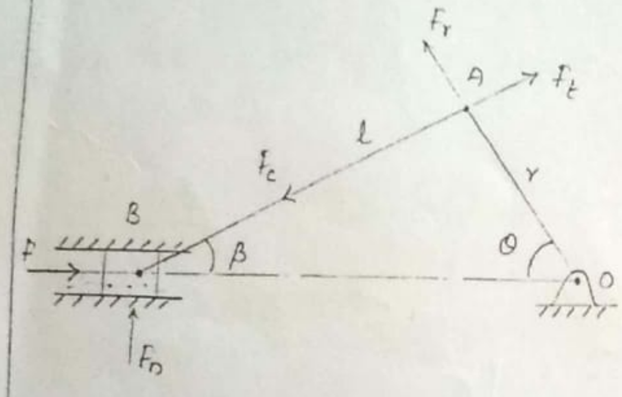
Dynamic force analysis: Inertia force and inertia torque. D'Alemberts principle, analysis of mechanisms (four bar linkages only),

Force Analysis of spur- helical - bevel and worm gearing

Module. 2

Dynamic force analysis

Slider crank mechanism



Displacement, Velocity and acceleration of piston

disp.
$$x_p = r(1-\cos\theta)$$
vel.
$$v_p = r\omega\left[\sin\theta + \frac{\sin 2\theta}{2n}\right]$$
acc
$$a_p = r\omega^2\left[\cos\theta + \frac{\cos 2\theta}{n}\right]$$
$$n = \frac{l}{r}$$

Angular vel., ang. acc. of connecting rod.

$$\omega_c = \omega \times \frac{\cos\theta}{\sqrt{n^2 - \sin^2\theta}}$$
$$\alpha_c = -\omega^2 \sin\theta\left[\frac{n^2-1}{(n^2-\sin^2\theta)^{3/2}}\right]$$

Engine force analysis

1. Piston effort [effective driving force]

Piston effort (P.E) F = $F_p - F_i$ (horizontal engines)

where F_p = Press. force = $P_1A_1 - P_2A_2$ (Covered - rod end)

F_i = Inertia force = $m \times a$.

Vertical $F = F_p - F_i + mg$

2. Thrust force along connecting rod. (F_c)

$$F_c = \frac{F}{\cos\beta}$$

3. Thrust force on sides of cylinder (F_n)

$$F_n = F_c \times \sin\beta$$
$$= F \times \tan\beta$$

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4. Crank pin effort (F_t)

It is the net force acting ⊥' to the crank

$$F_t = F_c \times \sin(\theta+\beta)$$

5. Thrust force on the bearings (F_r)

It is the radial force along crank

$$F_r = F_c \times \cos(\theta+\beta)$$

Turning moment on crankshaft.

Crank moment $T = F_t \times r$.

* A horizontal gas engine running at 210 rpm has a bore of 220mm and a stroke of 440mm. Connecting rod is 924mm long and reciprocating parts weight 20kg. When crank has turned through an angle of 30° from the inner dead centre. Gas press. on cover and crank side are 500 kN/m^2 and 60 kN/m^2 resp. Dia of piston rod is 40mm. Determine

a) T.M of crank shaft

b) Thrust force on bearings

c) Acceleration of flywheel which has a mass of 8kg and radius of gyration is 600 mm when power of engine is 22 kW

Given datas

Speed $N = 210$ rpm $\Rightarrow \omega = \frac{2\pi N}{60} = \frac{2\pi \times 210}{60}$

$= 21.99$ rad/sec

Stroke $L = 440$ mm

$L \Rightarrow L = 2r$

$r = 220$ mm $= 0.22$ m

Connecting rod length, $l = 924$ mm

$= 0.924$ m

reciprocating part weight, $m = 20$kg

θ, Crank angle $= 30°$

$P_1 = 500\ kN/m^2 = 500 \times 10^3\ N/m^2$

$P_2 = 60\ kN/m^2 = 60 \times 10^3\ N/m^2$

$D = 220$ mm $= 0.22$ m

$d = 40$ mm $= 0.04$ m

c) T.M of crank

$T = F_t \times r$

$F_t = F_c \times (\sin(\theta+\beta))$

$F_c = \dfrac{F}{\cos\beta}$

$F = F_p - F_i$

$F_p = P_1A_1 - P_2A_2$ $\qquad F_i = m \times a$

$= (500\times10^3)\times\frac{\pi}{4}(.22)^2 - (60\times10^3)\times\frac{\pi}{4}(0.22^2 - 0.04^2)$

$= 16801.2375$ ~~[illegible]~~ N

$F_i = m \times a$ $\qquad a = r\omega^2\left(\cos\theta + \dfrac{\cos 2\theta}{n}\right)$

$= 20 \times 104.7952$ $\qquad n = \dfrac{l}{r} = \dfrac{0.924}{0.22} = 4.2$

$= 2095.9048$ N $\qquad a = (0.22)\times(21.99)^2 \times \left(\cos 30 + \dfrac{\cos 60}{4.2}\right)$

$= \cancel{40.526}\ 104.7952$ m/s

$\therefore F = 16801.2375 - 2095.9048$

$= 14705.3326$ N.

$F_c = \dfrac{F}{\cos\beta}$

$= \dfrac{14705.3326}{\cos 6.837}$

$= 14810.6535$ N.

Sine rule $\Rightarrow$

$\dfrac{l}{\sin\theta} = \dfrac{r}{\sin\beta}$.

$\dfrac{0.924}{\sin 30} = \dfrac{0.22}{\sin\beta}$

$\sin\beta = 0.119$

$\beta = 6.837$

$F_t = F_c \sin(\theta+\beta)$

$= 14810.6535 \sin(30 + 6.837)$

$= 8879.5875$ N

$T = F_t \times r = 8879.5875 \times 220 \times 10^{-3}$

$= 1.9535 \times 10^3$ Nm

b) $F_r = F_c \cos(\theta+\beta)$

$= 14810.6535 \times \cos(30 + 6.837)$

$= 11853.6232$ N.

~~c) We have~~

c) We have the relation

acceleration torque = Turning moment − Resisting torque

$I_c \times \alpha = 1953.0385 - T_r$

$2.88 \times \alpha = 1953.0385 - 1000 \Rightarrow$

$\alpha = \frac{953.038}{2.88}$

$= 330.9159$ rad/sec²

Power, $P = T_r \times \omega$

$22 \times 10^3 = T_r \times 21.99$

$T_r = 1000$

$I = mk^2$

$= 8 \times (600 \times 10^{-3})^2$

$= 2.88$

* Crank and connecting rod of a vertical petrol engine running at 1800 rpm are 60mm and 270 mm resp. the dia of piston is 100mm and the mass of reciprocating engine parts is 1.2 kg. During the expansion stroke when the crank has turned 20° the gas press is 650 kN/m². Find

a) Net force on the piston
b) Net load on the gudgeon pin
c) Thrust force on cylinder wall
d) Speed at which the gudgeon pin load is reversed in direction

Given datas

$N = 1800$ rpm $\Rightarrow \omega = \frac{2\pi N}{60} = \frac{2\pi \times 1800}{60}$

$= 188.4955$ rad/sec

$r = 60$ mm
$= 0.06$ m
$l = 270$ mm
$= 0.27$ m

$n = \frac{l}{r} = \frac{0.27}{0.06} = 4.5$

$P_1 = 650 \times 10^3$ N/m² D = 0.1 m

a) $F_L = F_p - F_i + mg$

$F_p = P_1 A_1 - P_2 A_2$

$= (650 \times 10^3) \times \frac{\pi}{4} (0.1)^2$

$= 5105.088$ N

$F_i = m \times a$

$a = r\omega^2 \left[\cos\theta + \frac{\cos 2\theta}{n}\right]$

$= 0.06 \times (188.4955)^2 \left[\cos 20 + \frac{\cos 40}{4.5}\right]$

$= \cancel{1640.3645}$ 2366.1744 N

$F_i = m \times a$

$= \cancel{m} \; 1.2 \times 2366.1744$

$= 2839.4093\ N$

$mg = 1.2 \times 9.81$

$= 11.772\ N$

$\therefore F = F_p - F_i + mg$

$= 5105.088 - 2839.4093 + 11.772$

$= 2277.4561\ N$

b) Thrust on connecting rod / gudgeon pin

$F_c = \dfrac{F}{\cos\beta}$

$= \dfrac{2277.4561}{\cos(4.3589)}$

$= 2284.0627\ N$

Sine rule ⇒

$\dfrac{l}{\sin\theta} = \dfrac{r}{\sin\beta}$

$\dfrac{0.27}{\sin 20} = \dfrac{0.06}{\sin\beta}$

$\sin\beta = 0.076$

$\therefore \beta = 4.3589$

c) Thrust on cylinder wall

$F_n = F_c \times \sin\beta$

$= 2284.0627 \times 0.076$

$= 173.5887\ N$.

d) We have,

$F = F_p - F_i + mg$

Put $F = 0$

$0 = F_p - F_i + mg$

$\therefore F_i = F_p + mg$

$= 5105.088 + 11.772$

$= 5116.86\ N$

$F_i = m \times a$

$5116.86 = 1.2 \times (0.06)\, \omega^2 \left[\cos 20 + \dfrac{\cos 40}{4.5}\right]$

$\omega^2 = \dfrac{5116.86}{0.0799}$

$= 64040.801$

$\omega = 253.0628$ rad/sec

$N \Rightarrow \omega = \dfrac{2\pi N}{60}$

$\therefore N = \dfrac{\omega \times 60}{2\pi}$

$= \dfrac{253.0628 \times 60}{2\pi}$

$= 2416.5717$ rpm

In a vertical double acting steam engine the connecting rod is 4.5 times the crank radius. The weight of the reciprocating part is 120 kg and the stroke of the piston is 440 mm. The engine runs at 250 rpm. If the load on the piston due to pressure is 25 kN. When the crank has turned through an angle of 120°. Determine

a) Thrust in connecting rod
b) Press. on side wall
c) Tangential force on crank pin
d) Thrust on bearing
e) Turning moment.

Given datas.

$l = 4.5r \Rightarrow \frac{l}{r} = 4.5$ $\quad L = 440\,mm$

$r = 220 \times 10^{-3}\,m$ $\quad r = L/2 = 220\,mm$

$m = 120\,kg$

$N = 250\,rpm \Rightarrow \omega = \frac{2\pi N}{60} = \frac{2\pi \times 250}{60} = 26.1799\,rad/sec.$

$F_p = 25\,kN = 25 \times 10^3\,N.$

$\theta = 120°$

$a = r\omega^2\left(\cos\theta + \dots\right)$

$= 0.22 \times (26.1799)^2 \times \left(\cos 120 + \frac{\cos 240}{4.5}\right)$

$= -92.1465\,m/s$

a) $F = F_p - F_i + mg$

$F_i = m \times a$

$= 120 \times -92.1465$

$= -11057.57958\,N.$

$F = 25 \times 10^3 + 11057.5796 + (120 \times 9.81)$

$= 37234.7796\,N.$

$L = 4.5 \times 0.22$

Sine rule $\Rightarrow$

$\frac{l}{\sin\theta} = \frac{r}{\sin\beta}$

$\frac{0.99}{\sin 120} = \frac{0.22}{\sin\beta}$

$\sin\beta = 0.1925$

$\therefore \beta = 11.0958°$

b) $F_n = F_c \sin\beta$

$F_c = \frac{F}{\cos\beta}$

$= \frac{37234.7796}{\cos(11.0958)}$

$= 37944.0772\,N.$

c) $F_t = F_c \sin(\theta + \beta)$

$= 37944.0772 \sin(11.0958 + 120)$

$= 28595.0959\,N.$

d) $F_r = F_c \cos(\theta + \beta)$

$= 37944.07723 \cos(120 + 11.0958)$

$= -24941.4040\ 40103\ N$

i) $T = F_t \times r$

$= \cancel{36595.685}$

$\cancel{37944.0772} \times 0.22$

28595.0959

$= 6290.9211\ N/m$.

x. Crank and connecting rod of a vertical single cylinder gas engine running at 1800 rpm are 60 mm and 240 mm resp. Dia of piston is 80 mm mass of reciprocating part is 1.2 kg. During power stroke when piston has moved 20 mm from the TDC position, the pres. on the piston is 800 kN/m² find

1. Net force on the piston
2. Thrust in the connecting rod
3. Thrust on side of cylinder wall
4. Engine speed at which net force is zero

Given datas

$N = 1800\ rpm \Rightarrow \omega = \frac{2\pi N}{60} = \frac{2\pi \times 1800}{60} = 188.49$ rad/s

$r = 60\ mm = 0.06\ m$

$l = 240\ mm = 0.24\ m$

$d = 80\ mm = 0.08\ m$

$m = 1.2\ kg$

$P_1 = 800 \times 10^3\ N/m^2$

1 $F = F_p - F_i + mg$

$F_p = P_1 A_1$

$= 800\times10^3 \times \frac{\pi}{4}(0.08)^2$

$= 4021.2386\ N$

$F_i = m \times a$

$a = r\omega^2\left[\cos\theta + \frac{\cos 2\theta}{n}\right]$

$= 0.06 \times (188.49)^2 \left[\cos 48 + \frac{\cos 96}{4}\right]$

$= 1370.6855\ m/s^2$

$\therefore F_i = 1.2 \times 1370.6855$

$= 1644.82\ N.$

$n = \frac{l}{r} = \frac{0.24}{0.06} = 4$

$x = r(1-\cos\theta)$

given $x = 20\ mm$

$0.02 = 0.06(1-\cos\theta)$

$1-\cos\theta = 0.333$

$\cos\theta = 0.677$

$\theta = 47.9329 \simeq 48°$

$F = F_p - F_i + mg$

$= \cancel{1370}\ 4021.2386 - 1644.82 + (1.2\times9.81)$

$= 2388.1906\ N$

2. Thrust on connecting rod

$F_c = \frac{P}{\cos\beta} = \frac{2388.1906}{\cos 10.7} = 2430.449\ N$

Sine rule $\Rightarrow \frac{l}{\sin\theta} = \frac{r}{\sin\beta}$

$\sin\beta = \frac{\sin(48)\times0.06}{0.24}$

$= 0.1858$

$\beta = 10.7°$

3. $F_n = F_c \times \sin\beta$

$= 2430.449 \times 0.1858$

$= 451.577\ N.$

4. $F = 0 \Rightarrow F_p - F_i + mg = 0$

$F_i = F_p + mg$

$= 4021.2386 + (1.2\times9.81)$

$= 4033.0106\ N.$

$F_i = m \times a.$

$a = \frac{F_i}{m} = \frac{4033.0106}{1.2} = 3360.\cancel{84}\ 84\ m/s$

$a = r\omega^2\left[\cos\theta + \frac{\cos 2\theta}{n}\right]$

$\omega^2 \Rightarrow 3360.84 = 0.06 \times \omega^2\left[\cos 48 + \frac{\cos 96}{4}\right]$

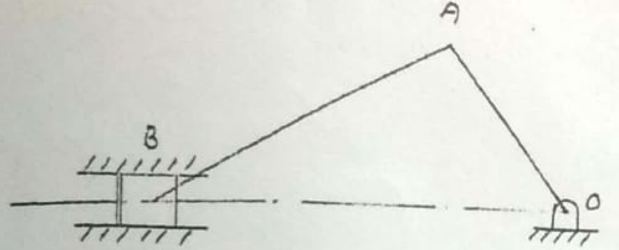
$\omega^2 = 87113.7348 \Rightarrow \omega = 295.150\ rad/s$

$N \Rightarrow \frac{2\pi N}{60} = \omega$

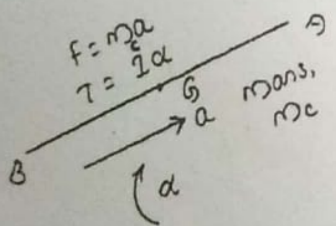
$N = \frac{\omega \times 60}{2\pi} = \frac{29515 \times 60}{2\pi}$

$= 2818.4783$ rpm.

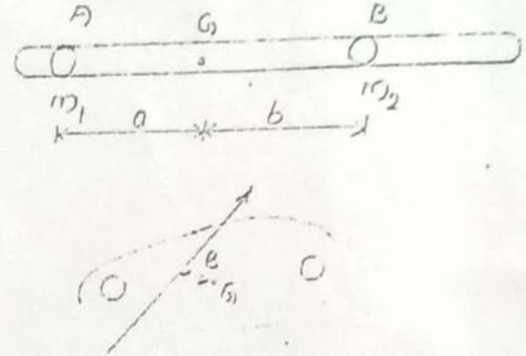
Dynamically equivalent System



Actual link.



Equivalent rigid link



$F = (m_1 + m_2)\, a$

$I = [(m_1 a^2) + (m_2 b^2)]\, \alpha$

Normally we consider only the masses of reciprocating parts in a slider crank mechanism. It is easy to solve the problems in dynamic analysis because the reciprocating part has only linear movement.

But when we consider mass of connecting rod into account the problem will be complicated b/c the centre of gravity is always changing due to combined effect of translation and rotation. In such cases an equivalent rigid link should consider by replacing the actual link.

The equivalent rigid link is defined as a link replaced for actual link which consists 2 point masses which has same force and couple of the actual link. In otherwords the linear acceleration and angular acceleration is same for link.

Take 1st condition ie; forces should be same.

$$F = m_c a \qquad F = (m_1 a + m_2 a)$$

$$m_c \alpha = (m_1 + m_2)\alpha$$

$$m_c = m_1 + m_2$$

Take 2nd condition ie; Torque should be same

$$I\alpha = (m_1 a^2 + m_2 b^2)\alpha$$

$$I = m_1 a^2 + m_2 b^2$$

Module 3

Flywheel analysis - balancing - static and dynamic balancing -balancing of masses rotating in several planes

Balancing of reciprocating masses - balancing of multi-cylinder in line engines - V engines - balancing of machines

Module: 3

Flywheel Analysis

It is a device used to store energy developed in engine

Let ω_1 be max speed of flywheel ω_2 be min speed of flywheel then;

ω = mean speed of flywheel

$$= \frac{\omega_1 + \omega_2}{2}.$$

Coefficient of fluctuation of speed (K)

$$K = \frac{\omega_1 - \omega_2}{\omega}.$$

max. fluctuation of energy. (e)

$$e = E_1 - E_2 \qquad E = \tfrac{1}{2} I \omega^2$$

$$= \tfrac{1}{2}(I\omega_1^2 - I\omega_2^2)$$

$$= \tfrac{1}{2} I (\omega_1^2 - \omega_2^2)$$

Relation b/w k and e

We have $e = \frac{1}{2} I (\omega_1^2 - \omega_2^2)$

$= \frac{1}{2} I (\omega_1 - \omega_2)(\omega_1 + \omega_2)$

$= \frac{1}{2} I (\omega_1 - \omega_2)\, \omega$

Mul. and div. by ω

$e = I\omega(\omega_1 - \omega_2) \times \frac{\omega}{\omega}$

$e = I\omega^2 k$

$= \frac{2}{2} I\omega^2 k$

$= 2 \times \frac{1}{2} I\omega^2 \times k$

$e = 2 \times E \times k$

* A flywheel with a mass of 3000 kg has a radius of gyration of 1.6 m. Find energy stored in flywheel when its speed increases from 315 rpm. to 340 rpm.

Given datas

Mass, $m = 3000\,kg$

Radius of gyration, $k = 1.6\,m$

Max. speed $N_1 = 340$ rpm

Min. speed $N_2 = 315$ rpm

Mean speed $N = \frac{N_1 + N_2}{2} = \frac{340+315}{2}$

$= 327.5$ rpm

Energy stored in flywheel $e = \frac{1}{2} I (\omega_1^2 - \omega_2^2)$

$I = mk^2$

$= 3000 \times (1.6)^2$

$= 7680\ kgm^2$

$\omega_1 = \frac{2\pi N_1}{60}$

$= \frac{2\pi \times 340}{60}$

$= 35.6047$ rad/s.

$\omega_2 = \frac{2\pi N_2}{60}$

$= \frac{2\pi \times 315}{60}$

$= 32.9867$ rad/s

$e = \frac{1}{2} \times 7680 \times (35.6047^2 - 32.9867^2)$

$= 690064.2355\ J$

* A flywheel absorbs 24 kJ of energy on increasing it's speed from 210 - 214 rpm. Determine its kinetic energy at 250 rpm.

Given datas

Energy $e = 24\,kJ = 24 \times 10^3\,J$

Speed $N_1 = 214$ rpm

$N_2 = 210$ rpm

$\therefore e = \frac{1}{2} I (\omega_1^2 - \omega_2^2)$

$\omega_1 = \frac{2\pi N_1}{60} = \frac{2\pi \times 214}{60}$

$= 22.41$ rad/s

$\omega_2 = \frac{2\pi N_2}{60} = \frac{2\pi \times 210}{60}$

$= 21.99$ rad/s.

$\therefore 24 \times 10^3 = \frac{1}{2} (\cancel{mk^2}\ I) \times (22.41^2 - 21.99^2)$

$\cancel{mk^2} = \frac{24 \times 10^3 \times 2}{(22.41^2 - 21.99^2)}$

$I = \cancel{2095.7529}\ 2574.$ kgm^2

$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 250}{60}$

$= 26.1799$ rad/sec.

$\therefore$ Kinetic energy $= \frac{1}{2} I \omega^2$

$= \frac{1}{2} \times 2574 \times (26.1799)^2$

$= 882098.2801$ J

---

* A flywheel fitted to a steam engine has a mass of 800 kg. It's radius of gyration is 360 mm. Starting torque of engine is 580 Nm. and may be assumed const. Find kinetic energy of flywheel after 12 s

Given datas

Mass, $m = 800$ kg

Radius of gyration, $k = 360$ mm $= 0.36$ m

Starting torque, $T = 580$ Nm

$N_1 = 0 \quad \therefore \omega_1 = 0$

$T = I\alpha$

$I = mk^2$

$= 800 \times (0.36)^2$

$= 103.68$ kgm^2

$580 = 103.68 \times \alpha$

$\therefore \alpha = 5.59$ rad/s

$v = u + at$

lly $\omega_2 = \omega_1 + \alpha t$.

$= 0 + (5.59 \times 12)$

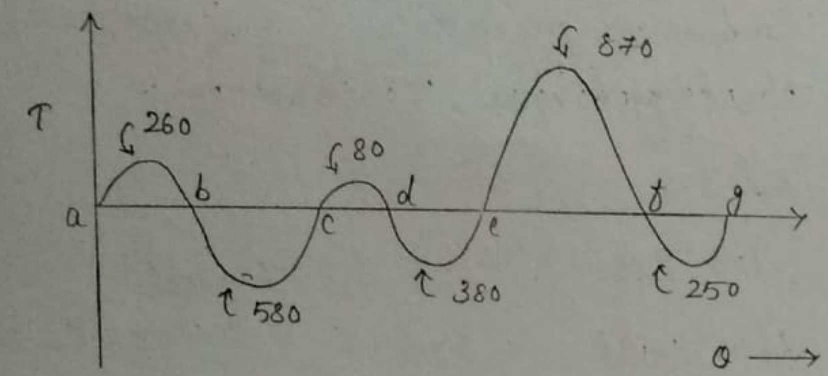
$= 67.1296$ rad/sec.

$e = \frac{1}{2} I \omega^2$

$= \frac{1}{2} \times 103.68 \times (67.1296)^2$

$= 233610.9049$ J

* The T-θ diagram for a petrol engine is drawn to vertical scale of 1mm = 500 Nm and horizontal scale of 1mm = 3°. Rotating part have a mass of 55 kg and radius of gyration of 2.1m. If engine speed is 1600 rpm. Determine coefficient of fluctuation of speed.



At a $e_a = E$

At b $e_b = E + 260$

At c $e_c = E - 320$

At d $e_d = E - 240$

At e $e_e = E - \cancel{380}\ 620$

At f $e_f = E + 250$

At g $e_g = E$

max. e − min e = e

$e = e_b - e_e$

$= E + 260 - E - 620$

$= 880\ mm^2$

Scale

$1mm = 500\ Nm$
$1mm = 3°\ Nm$ } $1mm^2 = \left(500 \times \frac{3\pi}{180}\right)$

$= 26.1799$

$\therefore e = 880 \times 26.1799$

$= 23038.3461\ J$

$mk^2 = 55 \times (2.1)^2$

$= 242.55\ kg\,m^2$

$N = 1600\ rpm \Rightarrow \omega = \frac{2\pi \times 1600}{60} = 167.5516\ rad/sec$

$\therefore \frac{1}{2} I\omega^2 = \frac{1}{2} \times 242.55 \times (167.5516)^2$

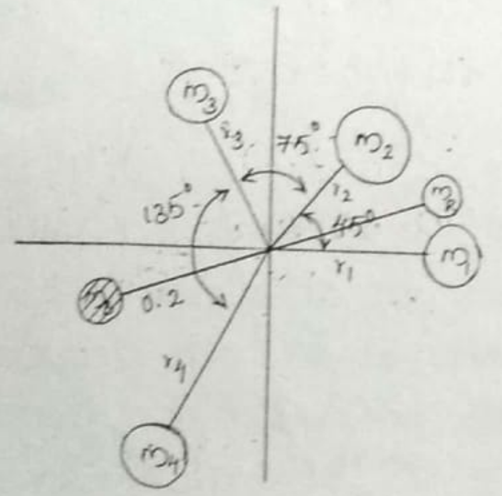
$= 3404618.401\ J$

$e = 2EK$

$K = \frac{e}{2E} = \frac{23038.3461}{2 \times 3404618.401} = 3.383 \times 10^{-3}$

Balancing of rotating masses.

* Four masses m_1, m_2, m_3 and m_4 are 200kg, 300kg, 240 kg and 260 kg resp. Corresponding radius of rotation are 0.2, 0.15, 0.25 and 0.3 m resp. Angle b/w successive masses are 45°, 75° and 135°. Find position and magnitude of balancing mass required if its radius of rotation is 0.2 m.



Given datas

$m_1 = 200$ kg $\quad m_4 = 260$ kg

$m_2 = 300$ kg

$m_3 = 240$ kg

$r_1 = 0.2$ m $\quad \theta_1 = 0°$

$r_2 = 0.15$ m $\quad \theta_2 = 45°$

$r_3 = 0.25$ m $\quad \theta_3 = 120°$

$r_4 = 0.3$ m $\quad \theta_4 = 255°$

Step. 1 $\sum F_H$

$$\sum F_H = F_{c_1}\cos\theta_1 + F_{c_2}\cos\theta_2 + F_{c_3}\cos\theta_3 + F_{c_4}\cos\theta_4$$

$F_{c_1} = m_1 r_1$

$$\therefore \sum F_H = m_1 r_1 \cos\theta_1 + m_2 r_2 \cos\theta_2 + m_3 r_3 \cos\theta_3 + m_4 r_4 \cos\theta_4$$

$$= (200 \times 0.2)\cos 0 + (300 \times 0.15)\cos(45) + (240 \times 0.25)\cos(120) + (260 \times 0.3)\cos(2$$

$$= 21.6319 \text{ N}$$

Step. 2 $\sum F_V$

$$\sum F_V = m_1 r_1 \sin\theta_1 + m_2 r_2 \sin\theta_2 + m_3 r_3 \sin\theta_3 + m_4 r_4 \sin\theta_4$$

$$= (200 \times 0.2)\sin 0 + (300 \times 0.15)\sin(45) \cdot (240 \times 0.25)\sin(120) + (260 \times 0.3)\sin(255)$$

$= 8.4391\ N.$

Step. 3 Resultant force

Resultant $F_R = \sqrt{\sum F_H^2 + \sum F_V^2}$

$= \sqrt{(21.6319)^2 + (8.4391)^2}$

$= 23.2197\ N$

Direction of resultant $\theta_R = \tan^{-1}\left(\frac{F_V}{F_H}\right)$

$= \tan^{-1}\left(\frac{8.4391}{21.6319}\right)$

$= 21.312^\circ$

Step. 4

$F_R = m_R \times r_R$

Given $r_R = 0.2\ m$

$m_R = \frac{F_R}{r_R}$

$= \frac{23.2197}{0.2} = 116.0985\ kg.$

So balancing mass $m_B = 116.0985$

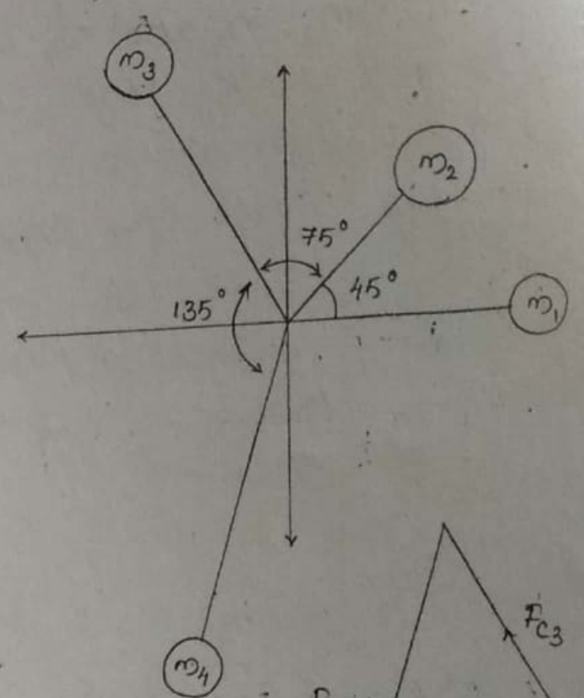
$\therefore$ direction of balancing mass

$\theta_B = \theta_R + 180$

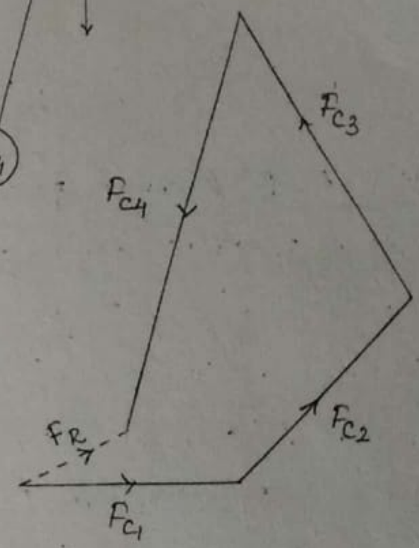
$= 201.31189^\circ$

Graphical method.

Scale
2cm = 0.1m



Scale
1cm = 10N



$F_R = 22\ N$

$\theta_R = 26^\circ$

$F_{C_1} = m_1 r_1 = 200 \times 0.2 = 40\,N$

$F_{C_2} = m_2 r_2 = 300 \times 0.15 = 45\,N$

$F_{C_3} = m_3 r_3 = 240 \times 0.25 = 60\,N$

$F_{C_4} = m_4 r_4 = 260 \times 0.3 = 78\,N$

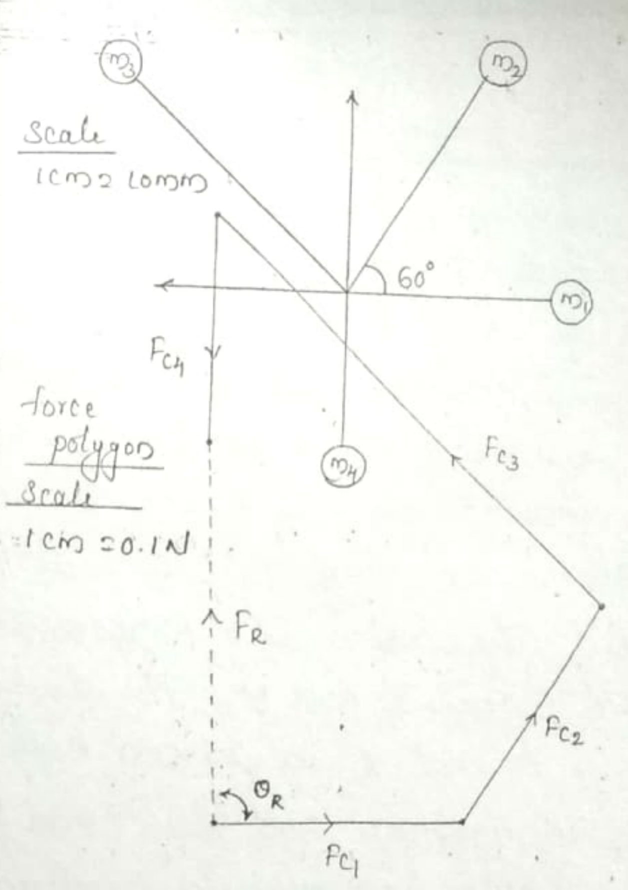
$\left.\begin{array}{l} F_R = 22\,N \\ \theta_R = 26^\circ \end{array}\right\}$ $F_R = m_R \times r_R$

$m_R = \frac{22}{0.2} = 110\,kg$

$\theta_B = 26 + 180$

$= 206^\circ$

* 4 masses A, B, C and D are attached to a shaft and revolve in the same plane. The masses are 12, 10, 18 and 15 kg resp. their radius of rotations are 40mm, 50mm, 60mm and 30mm. The angular position of masses B, C and D are 60°, 135° and 270° from mass A. Find mag. and pos. of balancing mass at a radius of 100mm.



$F_R = .75\,N$

$\theta_R = 90^\circ$

$m_R = \frac{F_R}{r_R} = \frac{.75}{0.1}$

$= 7.5\,kg$

$\therefore m_B = 7.5\,kg$

$\theta_B = 90 + 180 = 270^\circ$

Given data

$m_1 = 12\,kg$

$m_2 = 10\,kg$

$m_3 = 18\,kg$

$m_4 = 15\,kg$

$r_1 = 40\,mm$

$r_2 = 50\,mm$

$r_3 = 60\,mm$

$r_4 = 30\,mm$

$\theta_1 = 0^\circ$

$\theta_2 = 60^\circ$

$\theta_3 = 135^\circ$

$\theta_4 = 270^\circ$

$F_{C_1} = m_1 r_1$

$= 0.48\,\lambda$

$F_{C_2} = m_2 r_2$

$= 0.5\,\lambda$

$F_{C_3} = 1.08\,\lambda$

$= m_3 r_3$

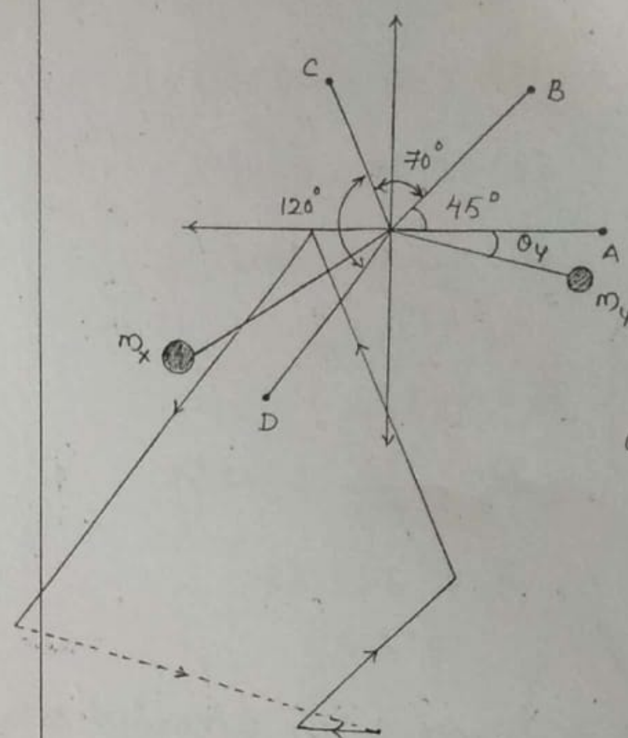
$F_{C_4} = m_4 r_4$

$= 0.45$

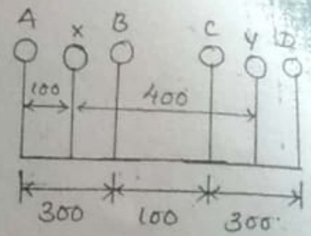
Balancing of rotating masses in different plane.

★ A shaft carries 4 masses - A, B, C and D of magnitude 200 kg, 300 kg, 400 kg and 200 kg resp. And revolving at radius 80 mm, 70 mm, 60 mm and 80 mm. In planes ~~mesh~~ measured from A at 300 mm, 400 mm and 700 mm. Angle b/w crank measured anticlockwise are A-B 45°, B-C 70° and C-D 120°. The balancing masses are placed in planes x and y. The distance b/w planes A and x is 100 mm and b/w x and y is 400 mm. and b/w y and D is 200 mm. If balancing masses revolves at a radius of 100 mm find their mag. and ang. position.

Side view



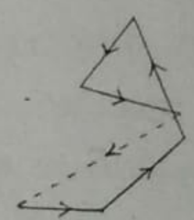
front view



$0.04\, m_y = 7.5$

$m_y = 187.5$ kg

$\theta_y = 14°$ [clockwise.



$0.1\, m_x = 36$

$m_x = 360$ kg

$\theta_x = 180 + 32.5$

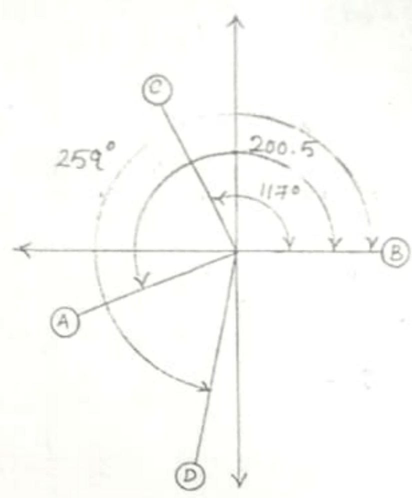
$= 212.5°$

(Anticlockwise)

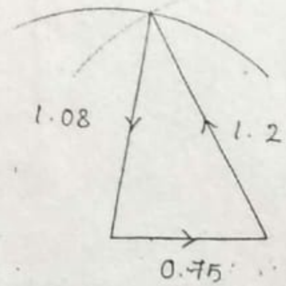
Plane	m Mass kg	r radius m	mr Force N	l Distance m	F×l Couple N-m
A	200	0.08	16	-0.1	-1.6
X (RP)	m_x	0.1	$0.1 m_x$	0	0
B	300	0.07	21	0.2	4.2
C	400	0.06	24	0.3	7.2
Y	m_y	0.1	18.75 $0.1 m_y$	0.4	$0.04\, m_y$
D	200	0.08	16	0.6	9.6

★ A, B, C and D are 4 masses carried by a rotating shaft at radius 100, 125, 200 and 150 ~~kg~~ mm resp. The planes in which the masses revolve are spaced 600 mm apart. And mass of B, C and D are 10 kg, 5 kg, and 4 kg resp. Find mass of ~~a~~ A and ang. position of all masses. So that the system remains in eqilibrium.

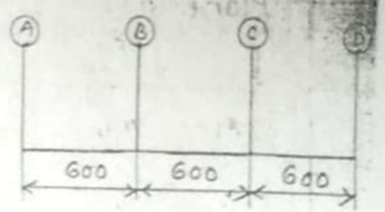
Side view



$\therefore \theta_A = 200.5^\circ$



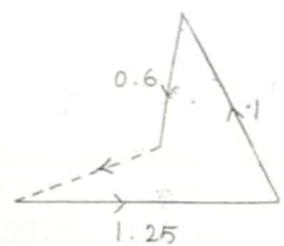
front view



$\theta_B = 0$

$\theta_C = 117^\circ$

$\theta_D = 259^\circ$



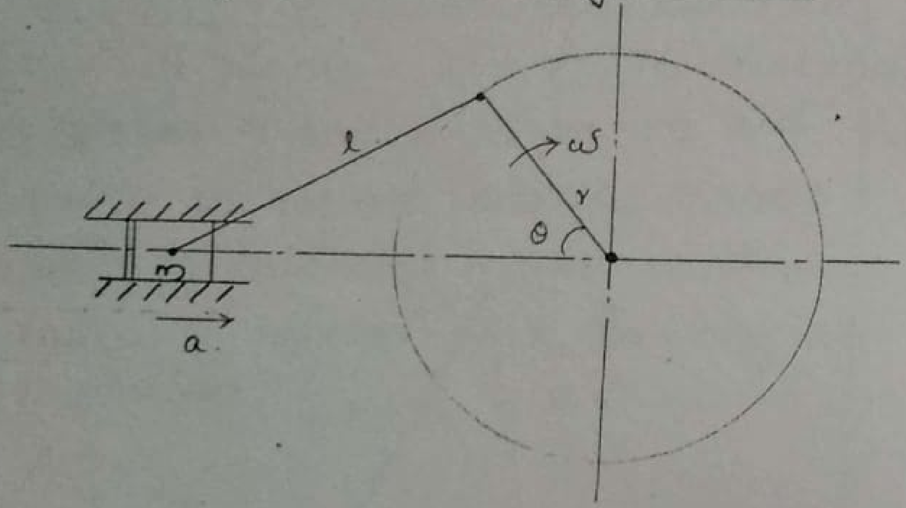
$0.1\, m_A = 0.75$

$\therefore m_A = 7.5$ kg.

$\theta_A = 200.5^\circ$

Plane	m mass kg	r radius m	mr Force N	l Distance m	F×l Couple N-m
A(RP)	m_A	0.1	$0.1 m_A$	0	0
B	10	0.125	1.25	$\frac{0.6}{600 \times 10^{-3}}$	0.75
C	5	0.2	1	1.2	1.2
D	4	0.15	0.6	1.8	1.08

Balancing of reciprocating masses



Force produced by the reciprocating mass

$f = m \times a$.

$$a = r\omega^2 \left[\cos\theta + \frac{\cos 2\theta}{n} \right]$$

$$\therefore f = m \times \left[r\omega^2 \left(\cos\theta + \frac{\cos 2\theta}{n} \right) \right]$$

$$= \underbrace{m r \omega^2 \cos\theta}_{\text{Primary un-balanced force}} + \underbrace{m r \omega^2 \frac{\cos 2\theta}{n}}_{\text{secondary un-balanced force.}}$$

Partial Balancing of 1° unbalanced force.

Step. 1

Convert ~~rotating~~ reciprocating mass m into rotating mass cm.

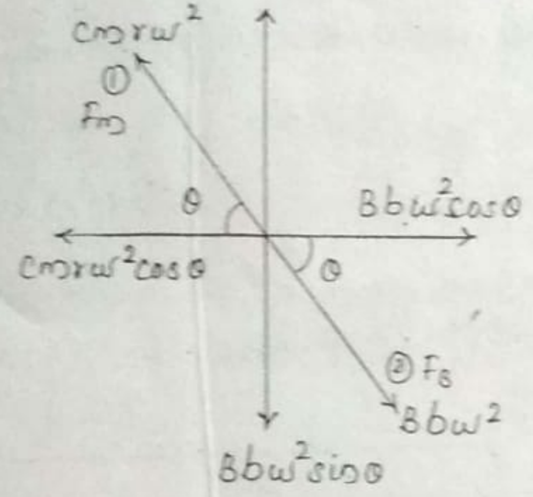
for converting the mass multiply with a factor c. So rotating mass = cm.

Step. 2

Balance the rotating mass cm by providing a balancing mass B at a radi 'b' opp. to the crank

Step. 3

Since m is reciprocating mass it ha only horizontal components. Since B is rotating mass it has 2 components of forces.



For balancing $F_m = F_B$

$$Cmr\cancel{\omega^2} = Bb\cancel{\omega^2}$$

$$Cmr = Bb.$$

Unbalanced force in x direction $= F_{UH}$

$$(1-C)(mr\omega^2\cos\theta)$$

Unbalanced force in y direction $= F_{UV}$

$$C \times (mr\omega^2 \sin\theta)$$

∴ Resultant ~~aba~~ unbalanced force =

$$\sqrt{F_{UH}^2 + F_{UV}^2}$$

Note:

$$B \times b = Cmr$$
$$= C(m + m_1)r$$

where m_1 is the mass of crank.

* A single cylinder reciprocating engine has speed 240 rpm. stroke 300 mm. Mass of reciprocating parts 50 kg. Mass of revolving parts at 150 mm radius 37 kg. If 2/3rd of the reciprocating parts and all the revolving parts are to be balanced find

1. The balance mass required at a radius of 400 mm
2. The residual unbalanced force when crank has rotated 60° from inner dead centre.

Given

$N = 240$ rpm

$L = 300$ mm $= 0.3$ m

$m = 50$kg

$m_1 = 37$ kg

$c = 2/3$ $\quad b = 400$ mm $= 0.4$ m

$r = 150$ mm

$= 0.15$ m

$B = ?$

1. We have

$B \times b = c(m + m_1) r$ b/c of the mass of revolving part is to be balanced at 2/3.

$0.4\, B = (2/3 \times 50 + 37) \times 0.15$

$B = 26.375$ kg.

2.

Horizontal unbalanced force

$F_{UH} = (1-c) m r \omega^2 \cos\theta$

$\omega = \frac{2\pi N}{60}$ $\quad \theta = 60°$

$= \frac{2\pi \times 240}{60}$

$= 25.1327$ rad/sec.

$\therefore F_{UH} = (1 - 2/3)\, 50 \times 0.15 \times (25.1327)^2 \times \cos 60$

$= 789.5684$ N

Vertical unbalanced force

$F_{UV} = c m r \omega^2 \sin\theta$

$= 2/3 \times 50 \times 0.15 \times (25.1327)^2 \times \sin 60$

$= ~~1364.5680~~$ N.

~~237~~ 2735.1360

Resultant

$F_R = \sqrt{(789.5684)^2 + (2735.1360)^2}$

$= 2846.8206$ N.

Balancing of inline engines

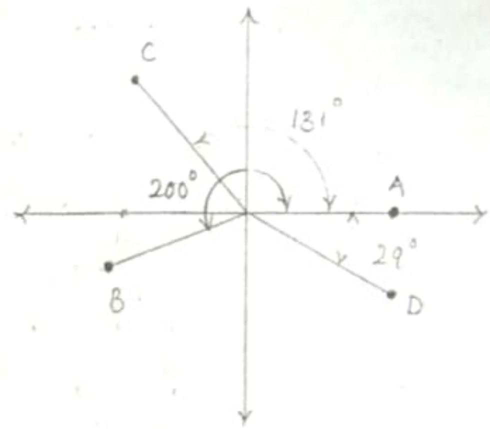
The multi cilinder engine with cylinders centre line in the same plane and on the same side of centre line of crankshaft. are known as inline engines

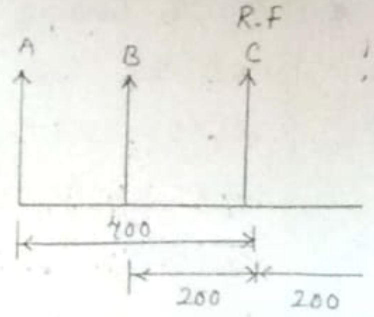
* A 4 cylinder vertical engine has crank 150 mm Long. The planes of rotation of 1st, 2nd and 4th cranks are 400, 200 and 200 mm resp. from 3rd crank

their and recipro

their reciprocating masses are 50, 60, and 50 kg resp. Find the mass of reciprocating part for 3rd cylinder. and relative ang. position of crank so that the engine may be complete balance

Plane	m Mass kg	r radius m	mr Force N	l Distance m	F×l Couple N m
A	50	.150	7.5	−0.4	−3
B	60	.150	9	−0.2	−1.8
C	m_c	.150	$0.15 m_c$	0	0
D	50	.150	7.5	0.2	1.5



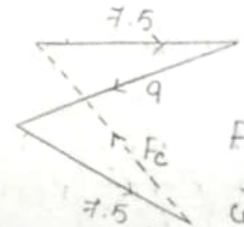


$\theta_A = 0$

$\theta_C = 131°$

$\theta_B = 200°$

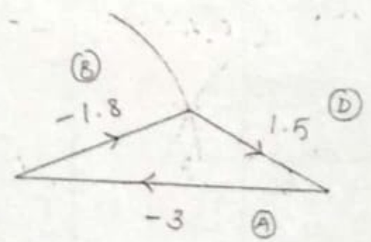
$\theta_D = 331°$



$F_c = 9N$

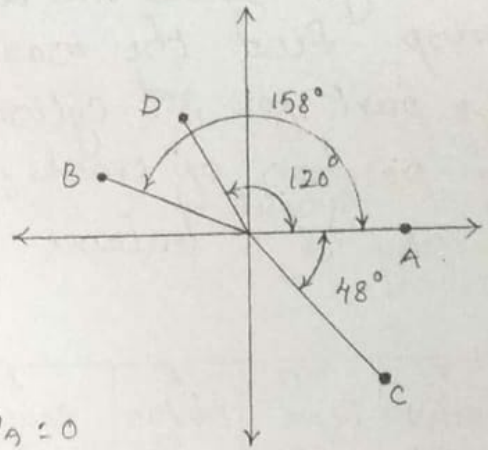
i.e; $0.15 m_c = 9$

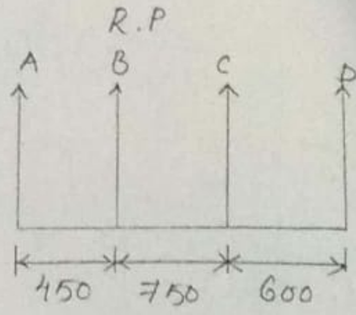
$m_c = 60 kg$



* A 4 crank engine has 2 outer crank set at 120° each other. and their corr. reciprocating masses are each 400 kg. The distance b/w the planes of rotation of adjacent crank are 450, 750, 600 mm. If the engine is to be in complete balance find reciprocate and ang. position of inner crank. If the length of each crank is 300 mm, length of each connecting rod is 1.2 m and speed of rotation is 240 rpm, what is the max. secondary unbalanced force.

Plane	m mass kg	r radius m	mr Force N	l Distance m	Fxl. Couple Nm
A	400	0.3	120	-0.450	-54
B	m_B	0.3	253.998 $0.3 m_B$	0	0
C	m_C	0.3	261.33 $0.3 m_C$	0.750	$0.225 m_C$
D	400	0.3	120	1.350	162



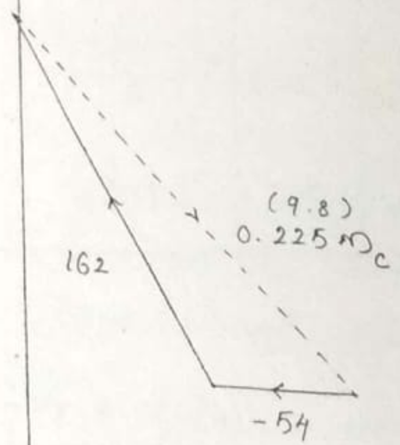


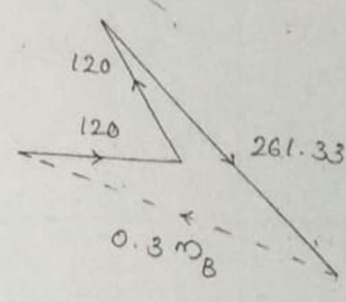
$\theta_A = 0$

$\theta_B = 158°$

$\theta_C = 312°$

$\theta_D = 120°$





$0.225\, m_C = 196$

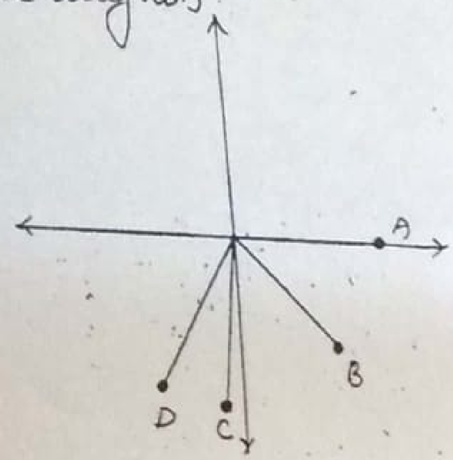
$m_C = 871.1$ kg

(6.35)

$0.3\, m_B = 254$

$m_B = 846.66$ kg

force diagram



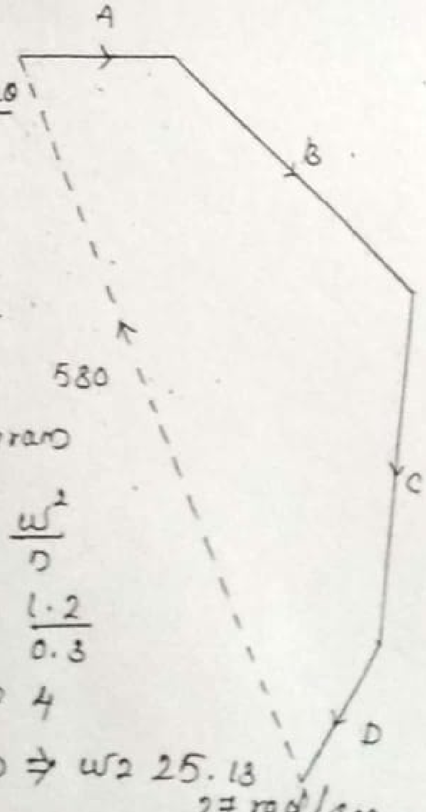
$\theta_A = 0$

$\theta_B = 316^\circ$

$\theta_C = 624^\circ$

$\theta_D = 240^\circ$

"since 2° force is obtained at crank angle 2θ"



$F_s = mr\omega^2 \frac{\cos 2\theta}{n}$

for max F_s

$\cos 2\theta = 1$

$F_s = mr\omega^2 \times \frac{1}{n}$

$mr = 580\ N$ from diagram

$\therefore F_s = 580 \times \frac{\omega^2}{n}$

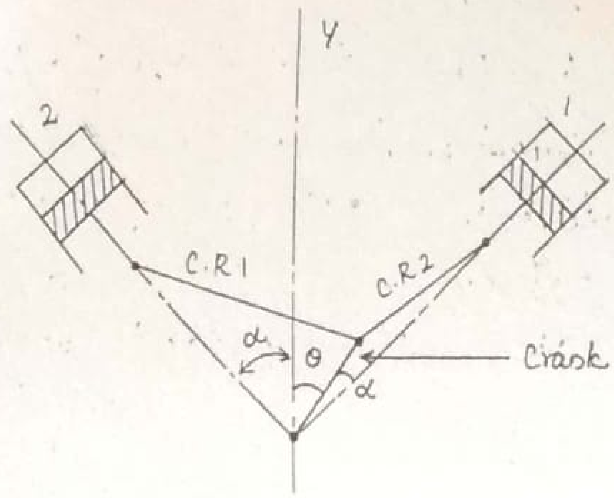
$n = \frac{l}{r} = \frac{1.2}{0.3}$

$= 4$

$N = 240$ rpm $\Rightarrow \omega = 25.13$ 27 rad/sec

$\therefore F_s =$ 91.5896 kN

Balancing of V-engine



Consider primary forces.

$F_{PH} = 2mr\omega^2 \sin^2\alpha \sin\theta$

$F_{PV} = 2mr\omega^2 \cos^2\alpha \cos\theta$

Note

At θ =

F_{PR} is

At θ =

F_{PR} is

Consider secondary forces

$F_{SH} = \frac{2mr\omega^2}{n} \sin\alpha \sin 2\alpha \sin 2\theta$

$F_{SV} = \frac{2mr\omega^2}{n} \cos\alpha \cos 2\alpha \cos 2\theta$

$F_{SR} = \sqrt{F_{SH}^2 + F_{SV}^2}$

Note

$\theta = 45^\circ$ F_{SR} is max.

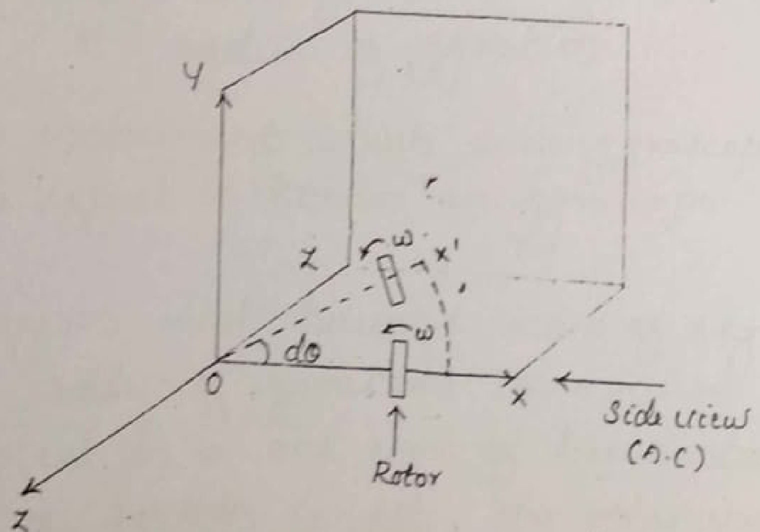
Module 4

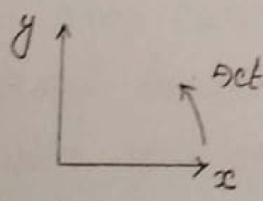
Gyroscope – gyroscopic couples Force Analysis of spur- helical - bevel and worm gearing

Gyroscopic action on vehicles - two wheelers, four wheelers, air planes and ships. Stability of an automobile – stability of a two wheel vehicle –Stabilization of ship.

Module 4

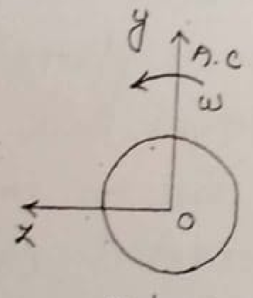
Gyroscopic couple





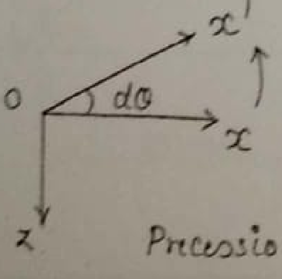
Front view.

Couple.



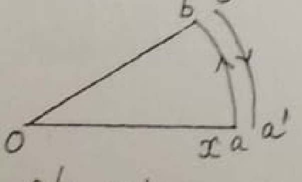
Side view

Spinning



Top view.

Precession



ab - active
b'a' - reactive

Consider a rotor is rotating in anticlock wise direction in the right side view. Therefore the axis ox is known as axis of spinning. The plane yz is known as plane of spinning.

Let ω be the angular speed of spinning.

Rotate this rotor in anticlockwise direction in top view at an angle dθ. So the new axis will be ox'. Axis is oy is known as axis of precession. Plane xz is known as plane of precession. Let ω_p be the angular speed of precession. Due to above two rotation the body is rotating clockwise in front view about about axis OZ. Therefore axis OZ is known as axis of gyroscopic couple. Plane xy is known as plane of gyroscopic couple.

According to Newton's 3rd Law, there will be a reactive couple and is known as reactive gyroscopic couple.

Gyroscopic Couple $C = I\omega\omega_p$

where $I = mk^2$

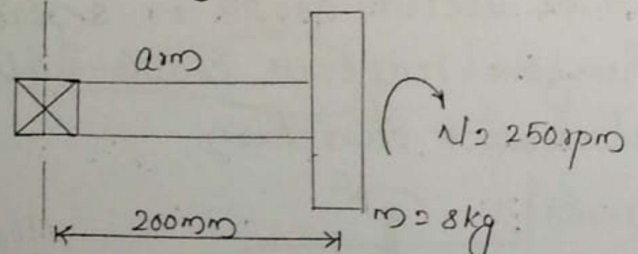
mass moment of inertia.

m - mass of rotor

k - radius of gyration

An instrument which uses gyroscopic couple effect is known as gyroscope.

* A uniform disk having mass of 8 kg and radius of gyration 150 mm is mounted on a one end of horizontal arm of 200 mm length, the other end of the arm can rotate freely in a universal bearing, the disk is given a clockwise spin of 250 rpm as seen from the disk end of the arm. Determine the motion of the disk.



Due to the weight of rotor the active gyroscopic couple is clockwise direction. To balance this couple a reaction gyroscopic couple act in anticlockwise direction.

$\therefore$ Couple C = weight $\times \perp^r$ distance.

$= mg \times d$

$= 8 \times 9.81 \times 0.2$

$= 15.696$ Nm.

Angular speed of spinning $\omega = \frac{2\pi N}{60}$

$= \frac{2\pi \times 250}{60}$

$= 26.18$ rad/sec.

Angular speed of precession ω_p

We have couple $C = I\omega\omega_p$

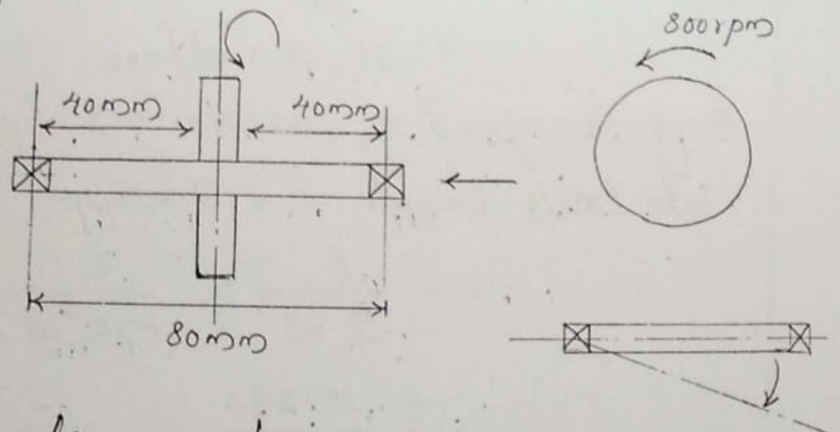
$\omega_p = \frac{C}{I\omega} = \frac{C}{mk^2\omega}$.

$= \frac{15.696}{8 \times 0.15^2 \times 26.18}$

$= 3.33$ rad/sec.

So the direction of precession is clockwise

* A disc with radius of gyration of 60mm and a mass of 4 kg. is mounted centrally on a horizontal axle of 80mm length b/w bearings. It spins about axle at 800 rpm counterclockwise, when viewed from RHS of bearing. The axial precession about vertical axis at 50 rpm is clockwise direction when viewed above. Determine the resultant reaction at each bearing due to mass and gyroscopic effect.



Angular speed of spinning $\omega = \frac{2\pi N}{60}$

$= -\frac{2\pi \times 800}{60} = -83.775$ rad/sec.

Angular speed of precession $\omega_p = \frac{2\pi}{60}$

$= \frac{2\pi}{60} \times 50$

$= 5.236$ rad/sec.

So resultant active gyroscopic couple is -ve, anticlockwise direction

$\therefore$ couple $C = I\omega\omega_p$

$= mk^2\omega\omega_p$

$= 4 \times 0.06^2 \times 5.236 \times -83.775$

$= -6.3165$ Nm.

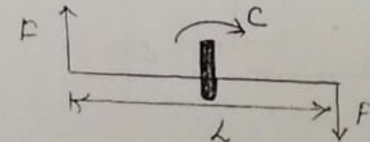
Couple due to mass.

weight of disk $= mg = 4 \times 9.81$

$= 39.24$ N.

Reaction at both side $R_m = 19.62$ N.

Since active couple is anticlockwise therefore reactive couple will be in clockwise direction.



$C = F \times L$

$= \cancel{19.62}\, F \times 0.08$

$F = \dfrac{C}{0.08} = \dfrac{6.316}{0.08} = 78.95\ N.$

$\therefore$ Total reaction at bearings

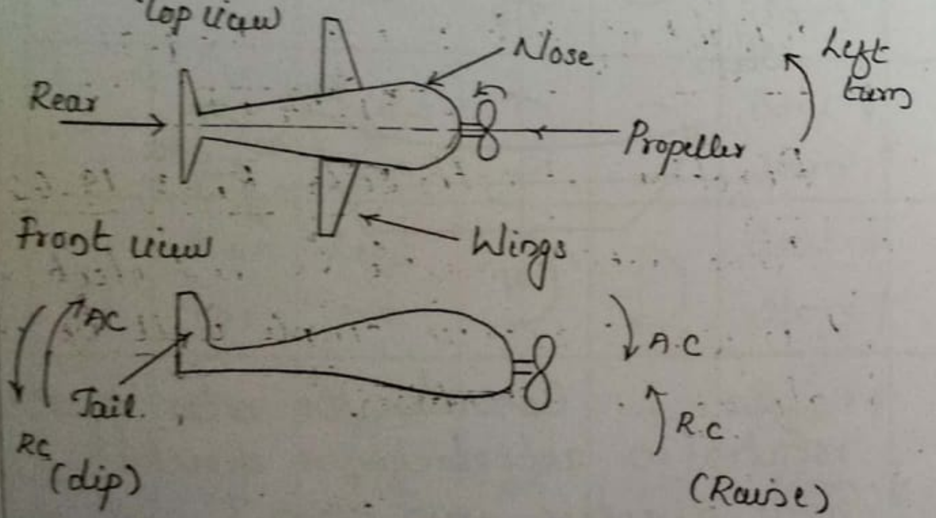
$R_A = R_D + F \qquad R_B = R_D - F$

$= 19.62 + 78.95 \qquad = 19.62 - 78.95$

$= 98.57\ N \qquad = -59.33\ N$

upwards downwards.

Effect of gyroscopic couple on aeroplane.



Consider the propeller of aeroplane rotating in anticlockwise direction [clockwise direction in rear view] with speed ω rad/s when the aeroplane takes left turn [anticlockwise precession] with speed ωp rad/sec. The active couple will be clockwise direction. Therefore reaction couple will be ~~+ve~~ -ve, anticlockwise direction. So it will raise nose and dip the tail of aeroplane.

Case	Spinning Propeller	Precession Turn	AGC	RGC	Effect
1	A.C -ve	Left -ve			Nose ↑ Tail ↓
2	A.C -ve	Right +ve			Nose ↓ Tail ↑
3	C.W +ve	Left -ve			Nose ↓ Tail ↑
4	C.W +ve	Right +ve			Nose ↑ Tail ↓

* An areoplane makes a complete half circle of 15 m radius towards left when flying at 200kmph. The rotory engine and propeller of plane has a mass of 400kg. and radius of gyration 0.3 m. Engine rotates at 2400 rpm clockwise when viewed from rear. Find gyroscopic couple, and its effect

Gyroscopic couple $C = I\omega\omega_p$

$I = mk^2$

$= 400 \times 0.3^2$

$= 36\ kg\,m^2$

$\omega = \frac{2\pi N}{60}$

$= \frac{2\pi \times 2400}{60}$

$= 251.3274\ rad/sec$

$\omega_p = \cancel{\frac{2\pi N}{60}}$

$\cancel{N = 200 \times \frac{5}{18}}$

$\cancel{= 55.55\ m/s}$

$v = 200 \times \frac{5}{18}$

$= 55.55\ m/s$

We have $v = r\omega_p$

$\therefore \omega_p = \frac{v}{r} = \frac{55.55}{15} = 3.703\ rad/sec.$

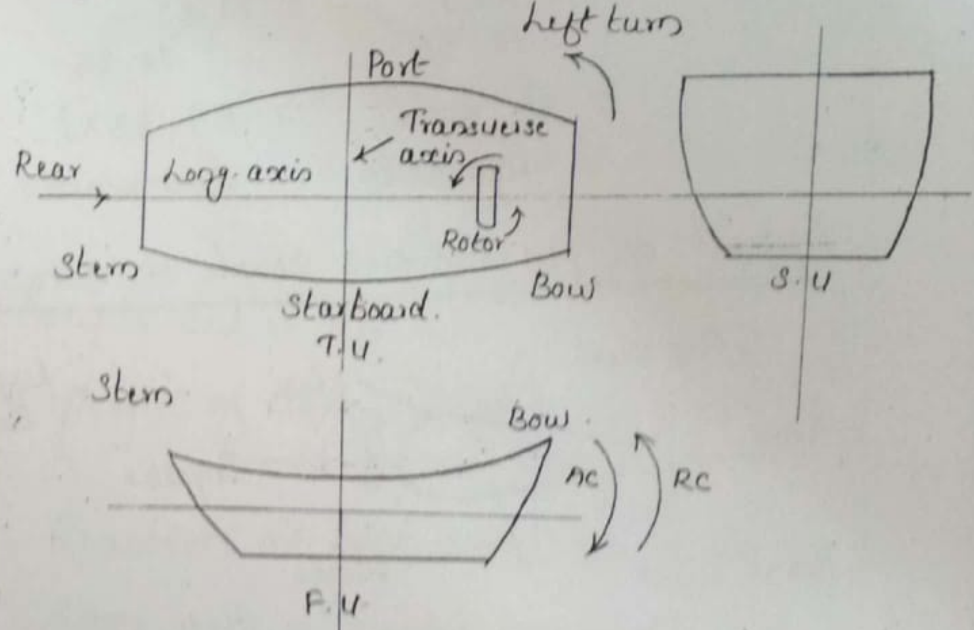
$C = 36 \times 3.703 \times 251.3274$

$= 33503.953\ Nm.$

Effect

Couple - clockwise
reaction - anticlockwise $\}$ Nose ↑ Tail ↓

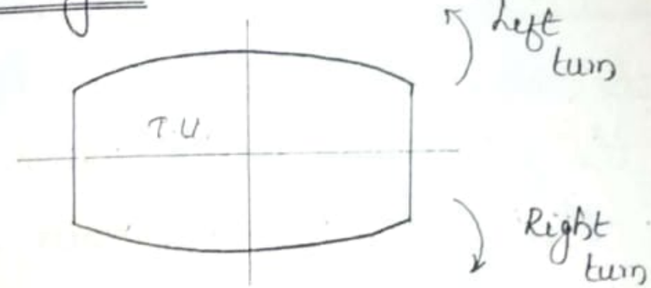
Effect of gyroscopic couple on ship



Consider the rotor of ship rotates in anticlockwise direction [clock wise direction from stern] with speed rad/sec. ~~The active couple will be clockwise direction~~ When the ~~aer~~ ship takes left turn [anticlockwise direction - precession] with speed up rad/sec. The active couple will be clockwise direction. Therefore reaction couple will be -ve, anticlockwise direction. So it will raise bow and dip the stern of the ship.

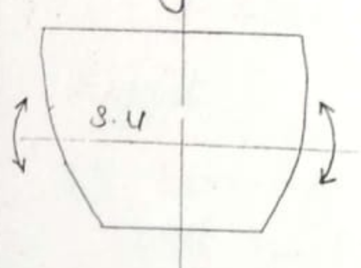
Case	Spinning Propeller	Precession Turn	A.G.C	R.G.C	Effect
1	A.C -ve	Left -ve	↻	↺	Bow ↑ Stern ↓
2	A.C -ve	Right +ve	↺	↻	Bow ↓ Stern ↑
3	C.W +ve	Left -ve	↺	↻	Bow ↓ Stern ↑
4	C.W +ve	Right +ve	↻	↺	Bow ↑ Stern ↓

Steering.



Turning of ship into left or right in the top view is known as steering

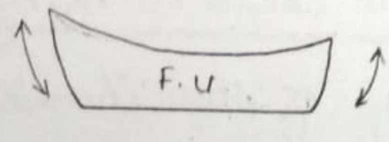
Rolling



Rotation of ship based on longitudin axis

Pitching.

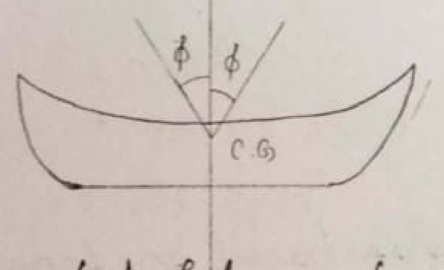
Rotation based on transverse axis



Effect of gyroscopic couple on Pitching.

If the rotor rotates at anticlockwise and the pitching is at upwards at Bow, then the ship will move towards right - starboard.

If rotor rotates at anticlockwise and pitching is downwards at Bow, then ship will move towards left - Port.



Let ϕ be angle of pitching, ω_{pit} be speed of pitching then speed of precession

$\omega_p = \omega_p \times \phi$ $\quad (\alpha)_{max} = \phi \times \omega_{pit}^2$

Effect of gyroscopic couple on rolling.

During rolling of ship the axis of spinning always const. Therefore there is no precession occurs. That is $\omega_p = 0$ implies Couple = 0.

2)

★ A ship propelled by turbine ~~rotate~~ rotor has a mass of 5 tonne and has a speed of 2100 rpm and has a radius of gyration of 0.5 m and rotates in clockwise direction when viewed from stern. Find the gyroscopic couple effect during the following conditions.

a) ship sails at a speed of 30 kmph and steers to left in a curve having 60 m radius

b) The ship pitches 6° above and 6° below the horizontal position the bow is descending with max. velocity. The time period is 20 sec. for SHM. Also calculate the max. ang. ~~positive~~ acc. of pitching

c) The ship rolls with an ang. vel. of 0.08 rad/sec

a) $C = I\omega\omega_p$

$I = mk^2$

$= \cancel{5}000 \times 0.5^2$

$= 1250 \ kgm^2$

$\omega_p = \frac{U}{r}$

$= \frac{5/18 \times 30}{60}$

$= 0.1389 \ rad/s$

$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 2100}{60}$

$= 219.\cancel{905}91 \ rad/sec.$

$C = 1250 \times 0.1389 \times 219.91$

$= 38181.8737 \ Nm.$

b) $\phi = 6^\circ$

$= 0.1047 \ rad.$

$\omega_p = \omega_{pit} \times \phi$

$\omega_{pit} = \frac{2\pi}{T} = \frac{2\pi}{20}$

$= 0.3142$

$\therefore \omega_p = 0.3142 \times 0.1047$

$= 0.0329 \ rad/sec.$

$C = 0.0329 \times 1250 \times 219.91$

$= \cancel{86358.455 \ Nm}.$

$= 9043.798 \ Nm.$

$(\alpha)_{max} = \phi \, \omega_{pit}^2$

$= 0.1047 \times (0.3142)^2$

$= 0.01 \ rad/sec^2$

c) $\omega_p = 0.03 \ rad/sec.$

$C = I\omega_p\omega$

$= 1250 \times 0.03 \times 219.91$

$= 8246.625 \ Nm.$

Effect

a) Turn lefts, active couple clockwise and reactive couple anticlockwise. So bow goes up and stern goes down

b) Rotor rotates anticlockwise, pitching downwards at bow and the ship will move to left side or port.

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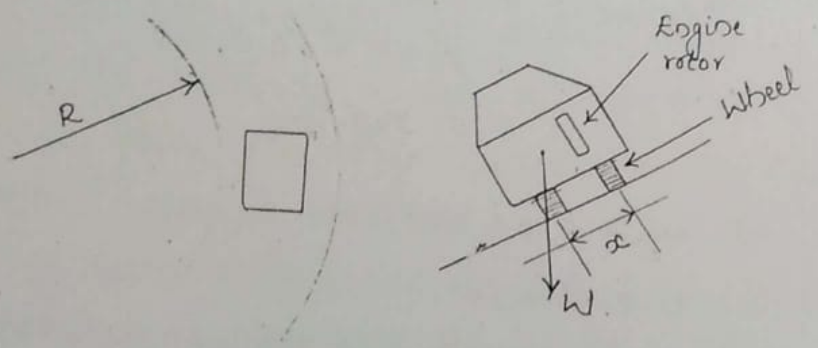
Stability of four wheel vehicle



When an automobile moves along a curved path we have to analyse 3 action

1. Weight of the automobile
2. Centrifugal action
3. Gyroscopic effect.

1. Weight of automobile

Let W be the weight of automobile which acts downwards. So the reaction will be vertically upwards.

$\therefore$ Reaction in each wheel $R = \frac{W}{4}$.

Centrifugal action.

When an automobile takes a turn a centrifugal force is developed to overturn the vehicle.

Centrifugal force $F_c = \frac{mv^2}{R}$.

$\therefore$ Centrifugal couple =

$$C_c = F_c \times h.$$

where h = Distance b/w CG of vehicle and road surface.

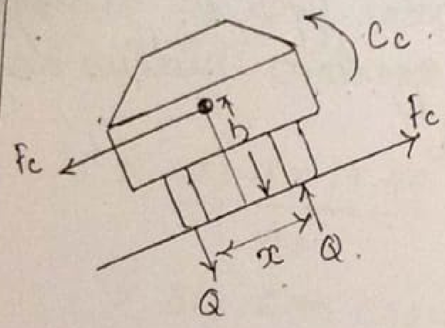
Let Q = reactive force on wheels then

$C_c = Q \times x$.

x - distance b/w two wheels.

Q will be vertically downwards in inner wheels and vertically upwards in outer wheels.

$\therefore$ The reaction developed on each wheel due to centrifugal action is $Q/2$



Gyroscopic couple of wheels

$C_w = 4 . I_w \omega_w \omega_p$

Gyroscopic couple for wheels Engine

$C_E = I_C \omega_C \omega_p$

∴ Total gyroscopic couple $C = C_w + C_E$

If wheel and engine have same direction

$C = C_w - C_E$

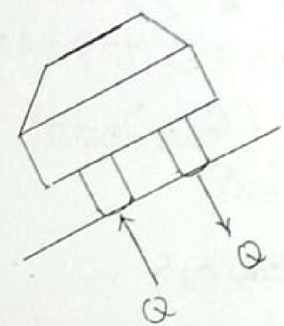
If they have diff direction

If C is +ve the direction of couple is as shown in fig.

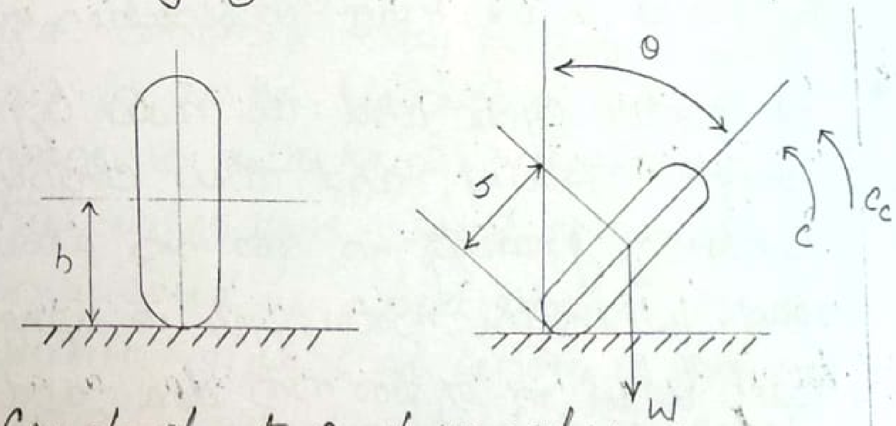
Couple $C = P \times x$

The reaction on each wheel due to gyroscopic effect is $P/2$.

When C is -ve.



Stability of two wheeler.



Couple due to centrifugal force.

$C_c = f_c \times h \cos\theta$

$= \frac{mu^2}{R} \times h \cos\theta \longrightarrow (1)$

Couple due to gyroscopic effect.

$C = C_W \pm C_E$

$= (2\, I_W\, \omega_W \omega_P \pm I_E\, \omega_E\, \omega_P) \cos\theta \longrightarrow (2)$

Couple due to weight (C_{we})

$C_{we} = w \times h \sin\theta$

$= mgh \sin\theta \longrightarrow (3)$

for the stability of 2 wheeler

$C_c + C = C_{we}$

Where θ is the max. inclination of heel.

* A motor cycle and its rider together weight 1700 N, and their combined centre of gravity is 500 mm above the road. When the motor cycle is upright. Each wheel is of 600 mm dia. and has a moment of inertia of $1.1\ kgm^2$. The moment of inertia of engine is $0.16\ kgm^2$. The engine rotates at 5 times speed of vehicle. Determine the angle of heel necessary when motor cycle is to turn over a track of 40m radius at speed of 70 kmph.

Given datas.

$W = 1700 N$

$h = 0.5 m$

$d_W = 0.6 m \Rightarrow r_W = 0.3 m$

$I_W = 1.1\ kg m^2$

$I_E = 0.16\ kg m^2$

$G = \frac{\omega_E}{\omega_W} = 5$

$R = 40 m$

$v = 70$ kmph

$= \cancel{37.08}$ m/s
19.44

Step. 1

Weight of the vehicle = 1700N

~~Reaction due to weight of the vehicle~~

$\cancel{R = \frac{W}{2} = 850 N}$

Step. 2

Couple due to centrifugal force

$C_c = \frac{mv^2}{R} \times h\cos\theta$

$= \frac{173.3 \times 19.44^2}{40} \times 0.5\cos\theta$

$= 818.655 \cos\theta$.

Step. 3

Couple due to gyroscopic couple.

$C = C_W + C_E$

$= [(2 I_W \omega_W \omega_P) + (I_E \omega_E \omega_P)] \cos\theta$

$\omega_W = \frac{v}{r_W} = \frac{19.44}{0.3}$

$= 64.8$ rad/sec.

$\omega_E = G \times \omega_W$

$= 5 \times 64.8$

$= 324$ rad/sec.

$\therefore C = (2 \times 1.1 \times 64.8 \times 0.486) + (0.16 \times 324 \times 0.486) \cos\theta$

$= 94.478 \cos\theta$.

$\omega_P = \frac{v}{R}$

$= \frac{19.44}{40}$

$= 0.486$ rad/sec.

$C_{we} = mgh\sin\theta$

$= 173.3 \times 9.81 \times 0.5 \sin\theta = 850.0365 \sin\theta$.

for stability of 2 wheeler

$C_c + C = C_{we}$.

$(818.655 + 94.478) \cos\theta = 850.0365 \sin\theta$.

$913.133 \cos\theta = 850.0365 \sin\theta$.

$\tan\theta = \frac{913.133}{850.0365}$

$= 1.0742$

$\therefore \theta = \tan^{-1}(1.0742)$

$= 47.0495^\circ$

* Find the angle of heel for the following data. Combined mass of vehicle with rider is 200 kg. Moment of inertia of engine is 0.25 kgm². Moment of inertia of wheel is 1.05 kgm². Speed of engine is 5 times speed of wheel and in the same direction. Height of centre of gravity is 0.55 m. 2 wheeler speed is 80 kmph. Wheel radius is 290 mm. Radius of turn is 60 m.

Given data

$W = mg = 1962\ N$

mass $m = 200 kg$

$I_E = 0.25\ kg m^2$

$I_W = 1.05\ kg m^2$

$G = \frac{\omega_E}{\omega_W} = 5$

$h = 0.55\ m$

$u = 80\ kmph$

$= 22.22\ m/s$.

$r_W = .29\ m$.

$R = 60\ m$.

Step. 1

Couple due to centrifugal force.

$C_c = \frac{m u^2}{R} \times h \cancel{\sin} \cos\theta$

$= \frac{200 \times 22.22^2}{60} \times 0.55 \cos\theta$

$= 905.1687 \cos\theta$.

Step. 2

Couple due to gyroscopic effect.

$C = C_W + C_E$

$= (2 . I_W \omega_W \omega_P) + (I_E \omega_E \omega_P)$

$\omega_W = \frac{u}{r_W} = \frac{22.22}{0.29} = 76.62\ rad/s$.

$\omega_E = G \times \omega_W = 5 \times 76.62 = 383.103\ rad/s$.

$\omega_P = \frac{u}{R} = \frac{22.22}{60} = 0.37\ rad/s$.

$C = [(2 \times 1.05 \times 76.62 \times 0.37) + (0.25 \times \overset{0.37}{\cancel{76.62}} \times 383.103)] \cos\theta$

$= 94.97 \cos\theta$.

Step. 3

Couple due to weight of the vehicle.

$C_{We} = mgh \sin\theta$

$= 1962 \times 0.55 \sin\theta$

$= 1079.1 \sin\theta$.

Step. 4

for stability of 2 wheeler

$\therefore C_{we} = C_c + C$

$1079.1 \sin\theta = (905.1687 + 94.97)\cos\theta$

$\tan\theta = \dfrac{\cancel{1079.1}\ \cancel{1000.1387}}{\cancel{1079}.\ \cancel{1000.1387}} \quad \dfrac{1000.1387}{1079.1}$

$= 0.9268$

$\theta = \tan^{-1}(0.9268)$

$= 42.825^\circ.$

Module 5

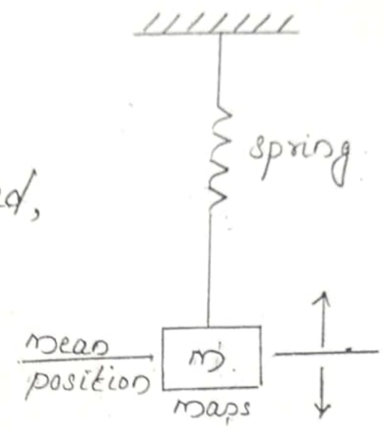
Introduction to vibrations – free vibrations of single degree freedom systems – energy Method

Undamped and damped free vibrations – viscous damping – critical damping - logarithmic decrement - Coulomb damping – harmonically excited vibrations

Response of undamped and damped system – beat phenomenon - transmissibility

Module. 5

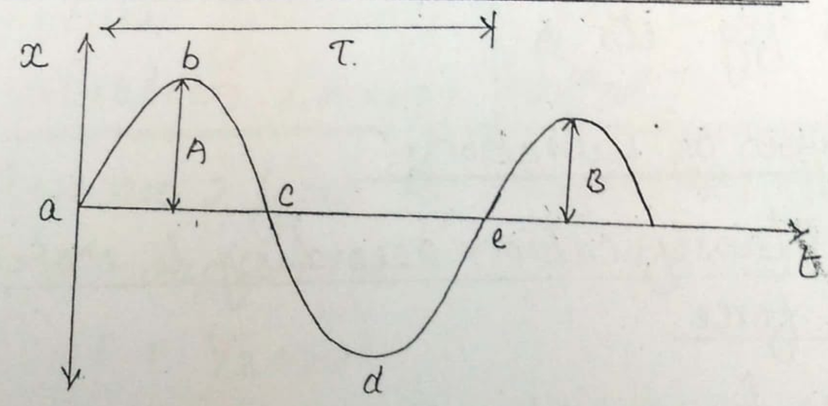
Vibration



When a body is displaced from its mean position and released, it executes a to and fro motion which is called vibration.

When the displacement occurs the spring is stored with strain energy during releasing of object this strain energy is converted into kinetic energy.

Common terms used in vibration



Normally vibration is represented by SHM or sine wave

Cycle

The motion which repeats after regular interval of time. is cycle.

Time period or Period of vibration (T)

Time taken to complete one cycle of vibration.

Frequency (f)

It is the no. cycles per second

$f = \frac{1}{T}$, Hz

Amplitude

The max. displacement of vibration In fig. its A.

Types of vibrations.

1. Classification according to external force.

a) free or natural vibration

The vibration occurs with the initial force and vibration cont with out external force is known free/natural vibration.

eg: Vibration of tuning fork
oscillation of ship.

b) Forced vibration

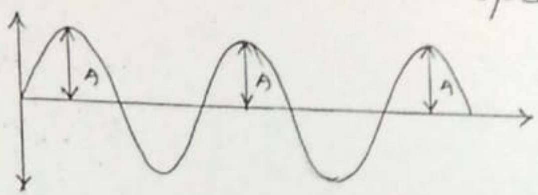
Vibration occurs continuously by the external force is forced vibration.

eg: Vibration of vehicle when engine works.
Vibration of machines.

2. According to amplitude.

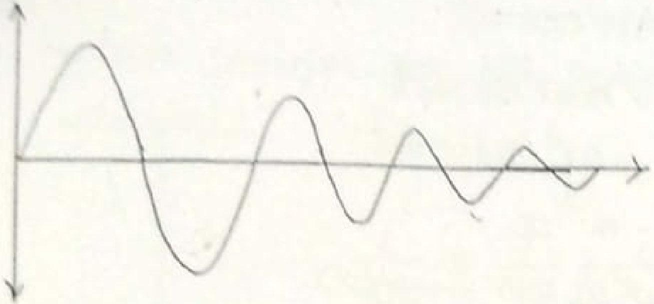
a) Undamped vibration

If the amplitude of vibration is const. then is called undamped vibratio



b) Damped vibration

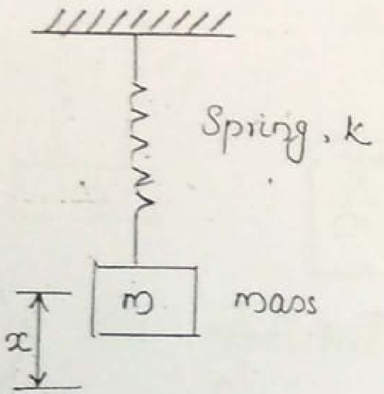
If the amplitude of vibration decreases with time is called damped vibration



3. According to Dof

1. Single dof
2. Two dof
3. Multiple dof

Free undamped single Dof vibration



The vibrating system consists of 2 parts mass, m and spring with stiffness k

There are 2 forcers developed in the system, inertia force due to mass

$F_i = ma$.

$= m\frac{d^2x}{dt^2}$

$= m\ddot{x}$

Spring force

We have stiffness = $\frac{\text{force}}{\text{deflection}}$

$k = \frac{f_s}{x}$

$\therefore f_s = kx$.

for eqbm, total forces must be zero

ie; $f_i + f_s = 0$

$m\ddot{x} + kx = 0$

This equation is known as equation of motion in vibration

Natural frequency of free undamped vibration [Energy method]

There are 2 types of energies in the system

1. Kinetic energy

   $KE = \frac{1}{2}mu^2$

2. Strain or Potential energy.

   $PE = \frac{1}{2}kx^2$

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According to energy method

Total energy = Const.

KE + PE = C

$$\frac{d}{dt}(KE) + \frac{d}{dt}(PE) = 0$$

$$\frac{d}{dt}\left(\tfrac{1}{2} m u^2\right) + \frac{d}{dt}\left(\tfrac{1}{2} k x^2\right) = 0.$$

$$\tfrac{1}{2}\frac{d}{dt}\left(m u^2 + k x^2\right) = 0$$

$$\frac{d}{dt}\left(m u^2 + k x^2\right) = 0.$$

$$\frac{d}{dt}\left(m\left(\frac{dx}{dt}\right)^2 + k x^2\right) = 0.$$

$$m \cdot 2\frac{dx}{dt} \cdot \frac{d^2x}{dt^2} + k\,2x \times \frac{dx}{dt} = 0$$

$$2\frac{dx}{dt}\left(m \cdot \frac{d^2x}{dt^2} + kx\right) = 0$$

$$m \cdot \frac{d^2x}{dt^2} + kx = 0$$

$$m \cdot \ddot{x} + kx = 0 \longrightarrow (1)$$

Since vibration is a SHM, Take ~~sea~~

$x = A \sin \omega t$

$\dot{x} = A\omega \cos \omega t$

$\ddot{x} = -A\omega^2 \sin \omega t$

$\quad = -\omega^2 A \sin \omega t$

$\ddot{x} = -\omega^2 x$

$\ddot{x} + \omega^2 x = 0 \longrightarrow (2)$

from eq (1) ÷ throughout by m

$$\ddot{x} + \frac{k}{m} x = 0 \longrightarrow (3)$$

Comparing (3) and (2)

$$\frac{k}{m}\cancel{x} = \omega^2 \cancel{x}$$

$$\omega^2 = \frac{k}{m}.$$

$$\therefore \omega = \pm\sqrt{\frac{k}{m}}$$

$$2\pi f_n = \pm\sqrt{\frac{k}{m}}$$

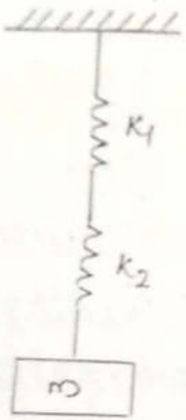
$$\boxed{\therefore f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}}$$

ie; Time period $T = \dfrac{1}{f_n}$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Case. 1

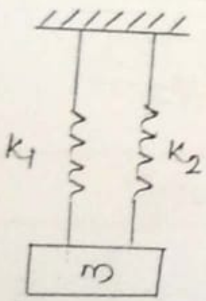
When springs are in series.



$$\frac{1}{K_{eq}} = \frac{1}{K_1} + \frac{1}{K_2}$$

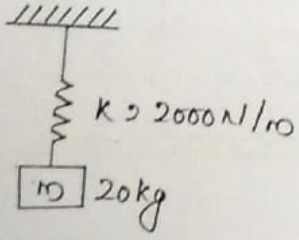
Case. 2

When springs are in parallel.



$$K_{eq} = K_1 + K_2.$$

* Find the natural frequency of the following.

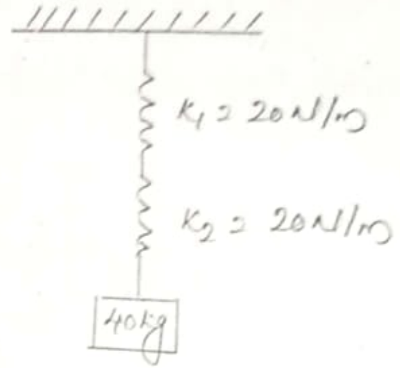


$$f_n = \frac{1}{2\pi}\sqrt{\frac{K_{eq}}{m}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{2000}{20}}$$

$f_n = 1.59$ Hz

b)



$$\frac{1}{K_{eq}} = \frac{1}{K_1} + \frac{1}{K_2}$$

$$= \frac{1}{20} + \frac{1}{20}$$

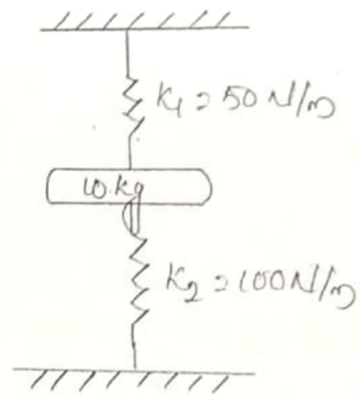
$\therefore K_{eq} = 10$ N/m

$$f_n = \frac{1}{2\pi}\sqrt{\frac{K_{eq}}{m}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{10}{40}}$$

$= 0.079$ Hz

c)



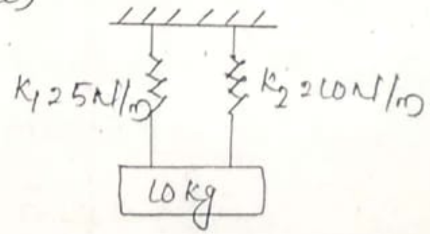
$K_{eq} = K_1 + K_2$

$= 50 + 100 = 150$ N/m

$$f_n = \frac{1}{2\pi}\sqrt{\frac{K_{eq}}{m}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{150}{10}} = 0.6 \text{ Hz}$$

d)



$K_{eq} = K_1 + K_2$

$= 5 + 10 = 15$ N/m

$$f_n = \frac{1}{2\pi}\sqrt{\frac{15}{10}}$$

$= 0.0616$ Hz.

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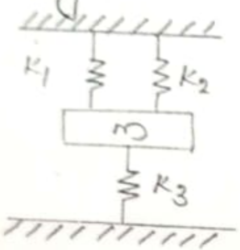
* Find the natural frequency of the system shown in fig. with $k_1 = 2000$ N/m, $k_2 = 2500$ N/m and $k_3 = 3000$ N/m. Mass $m = 5$ kg.



$K_{eq} = k_1 + k_2 + k_3$

$= 2000 + 2500 + 3000$

$= 7500$ N/m.

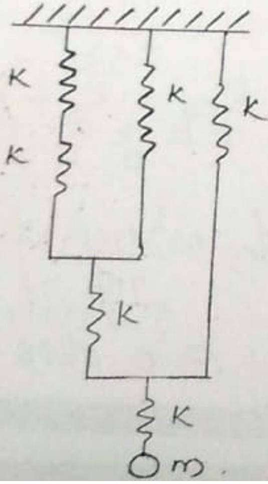
$$f_n = \frac{1}{2\pi}\sqrt{\frac{K_{eq}}{m}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{7500}{5}}$$

$$= 6.164 \text{ Hz}.$$

* $K = 5\times10^3$ N/m $m = 45$ kg.

Q)



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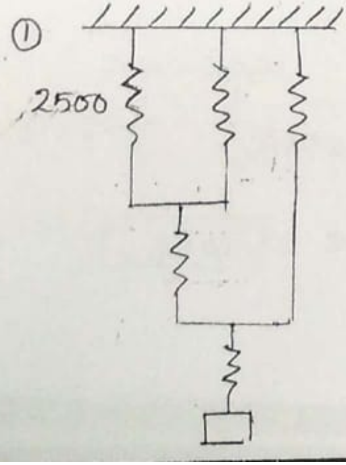

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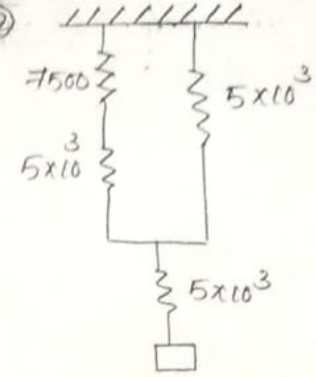

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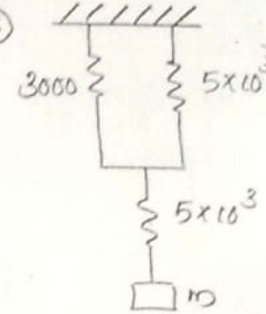

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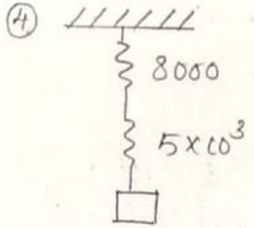


$K_{eq} = 3076.923$ N/m.

$$f_n = \frac{1}{2\pi}\sqrt{\frac{K_{eq}}{m}}$$

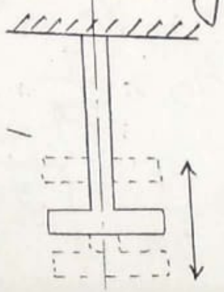
$$= \frac{1}{2\pi}\sqrt{\frac{3076.923}{45}}$$

$$= 1.316 \text{ Hz}.$$

Types of free vibration

1. Longitudinal

If vibration occurs along axis is known as longitudinal vibration.



Natural frequency is given by

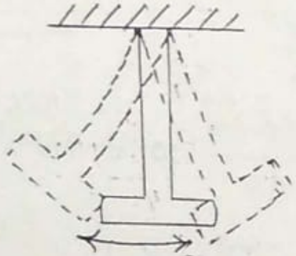
$$f_n = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}} \qquad k \times \delta = mg$$

$$\frac{k}{m} = \frac{g}{\delta}$$

where δ is $\frac{WL}{AE}$

2. Transverse vibration

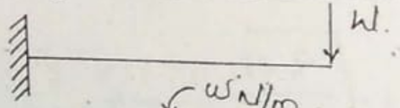
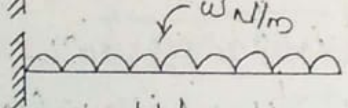
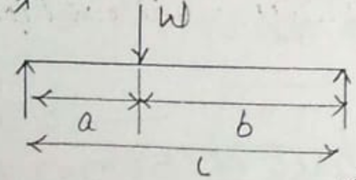
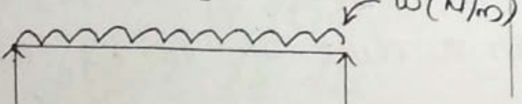
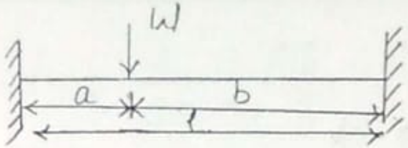
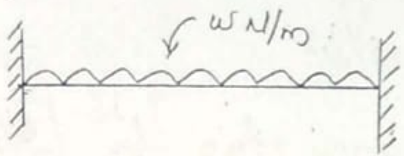
Vibration $\perp^r$ to the axis of shaft is known as transverse vibration.



Natural frequency

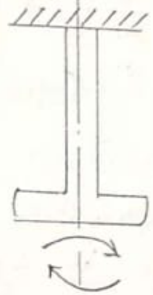
$$f_n = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}}$$

$$I = \frac{\pi d^4}{64}$$

Types of beam	Value of δ
① 	$\delta = \frac{WL^3}{3EI}$
② 	$\delta = \frac{WL^4}{8EI}$
③ 	$\delta = \frac{Wa^2b^2}{3EIL}$
④ 	$\delta = \frac{5WL^4}{384EI}$
⑤ 	$\delta = \frac{Wa^3b^3}{3EIL^3}$
⑥ 	$\delta = \frac{WL^4}{384EI}$

Torsional vibration

It is the vibration caused by twisting the axis of shaft



$$f_n = \frac{1}{2\pi}\sqrt{\frac{q}{I}}$$

q = stiffness of shaft

$$I = mk^2$$

$$= \frac{mR^2}{2}.$$

* A shaft of 100 mm dia and 1m long is fixed at one end and other end carries a flywheel of mass 1000kg. Take young's modulus as 200 GN/m². find natural frequency of longitudinal vibration.

Given datas

shaft dia d = 100mm

Length $L = 1m$

mass $m = 1000\ kg$

$E = 200 \times 10^9\ N/m^2$.

Natural frequency $f_n = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}}$.

$\delta = \frac{\cancel{Ax}\,WL}{AE}$

$W = 1000 \times 9.81$; $A = \frac{\pi}{4} d^2$

$= 9810\ N$

$= \frac{\pi}{4} \times (0.1)^2$

$= 7.85 \times 10^{-3}\ m^2$.

$\therefore \delta = \frac{9810 \times 1}{7.85 \times 10^{-3} \times 200 \times 10^9}$

$= 6.248 \times 10^{-6}\ m$.

ie; $f_n = \frac{1}{2\pi}\sqrt{\left(\frac{6.248 \times 10^{-6}}{9.81}\right)^{-1}}$

$= 199.427\ Hz$.

* A 40mm dia, 250mm long cantilever shaft has a disc of mass 75 kg at its free end. Young's modulus of shaft is $200\ GN/m^2$. Determine freq. of tran-sverse vibration

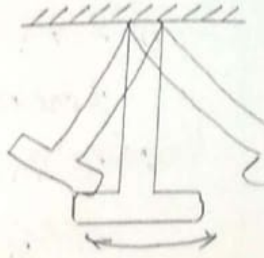
Given datas.

$d = 40mm$ $L = 0.25\ m$

$= 0.04\ m$

$m = 75\ kg \Rightarrow W = 735.75\ N$.

$E = 200 \times 10^9\ N/m^2$



$f_n = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}}$

$\delta = \frac{WL^3}{3EI}$ $I = \frac{\pi d^4}{64}$

$= \frac{\pi(\cancel{0.25}\ 0.04)^4}{64} = \cancel{1.9175}\ 1.2566 \times 10^{-7}\ m^4$

$= \frac{735.75 \times 0.25^3}{3 \times 200 \times 10^9 \times 1.2566 \times 10^{-7}}$

$= 1.5247 \times 10^{-4}\ m$.

$\therefore f_n = \frac{1}{2\pi}\sqrt{\frac{9.81}{1.5247 \times 10^{-4}}}$

$= 40.37\ Hz$.

* A cantilever shaft of 50mm dia 300mm long has a disc of 100 kg attached to

free end

Determin

free end. Young's modulus is 200 GN/m^2
Determine a) freq. of longitudinal vibration
b) freq. of transverse vibration

Given datas.

$d = 50\ mm$ $\quad E = 200 \times 10^9\ N/m^2$
$= 0.05\ m$

$L = 300\ mm$
$= 0.3\ m$

$m = 100\ kg \Rightarrow W = 981\ N$.

a) $f_n = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}}$ $\quad A = \frac{\pi}{4}(0.05)^2$
$= 0.001963\ m^2$
1.963×10^{-3}

$\delta = \frac{WL}{AE}$

$= \frac{981 \times 0.3}{0.001963 \times 200 \times 10^9}$

$= 7.4943 \times 10^{-7}\ m$.

$f_n = \frac{1}{2\pi}\sqrt{\frac{9.81}{7.4943 \times 10^{-7}}}$

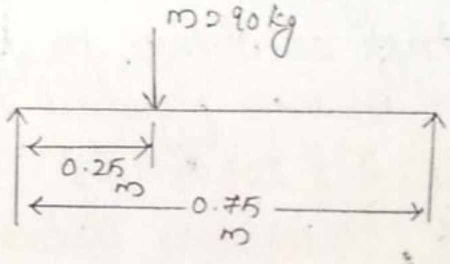
$= 575.82\ Hz$

b) $f_n = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}}$

$\delta = \frac{WL^3}{3EI}$ $\quad I = \frac{\pi d^4}{64}$
$= \frac{\pi}{64}(0.05)^4$
$=$ $\quad = 3.0679 \times 10^{-7}\ m^4$

☆ A shaft of length 0.75 m supported freely at 2 ends, it carries a body of mass 90 kg at 0.25 m from one end find natural freq. of transverse vibration assume E = 200 GN/m^2 and dia of shaft is 50 mm.

Given datas



$l = 0.75\ m$
$m = 90\ kg$
$W = 882.9\ N$.
$a = 0.25 \Rightarrow b = 0.5\ m$.
$E = 200 \times 10^9\ N/m^2$
$d = 0.05\ m$.

$$f_n = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}}$$

$$\delta = \frac{Wa^2b^2}{3EIL} \qquad I = \frac{\pi}{64}d^4 = \frac{\pi}{64}(0.05)^4 = 3.0679\times10^{-7}\ m^4$$

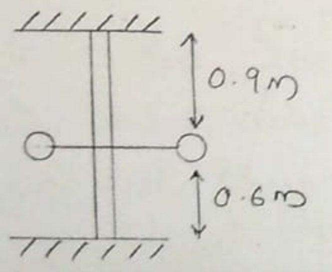
$$= \frac{882.9\times0.25^2\times0.5^2}{3\times200\times10^9\times3.0679\times10^{-7}\times0.75}$$

$$= 9.99\times10^{-5}\ m.$$

$$\therefore f_n = \frac{1}{2\pi}\sqrt{\frac{9.81}{9.99\times10^{-5}}}$$

$$= 49.867\ Hz.$$

* A flywheel is mounted on a vertical shaft as shown in fig. dia of shaft is 50 mm flywheel has a mass of 500 kg, E = 200 GN/m². find natural freq. of transverse vibration.



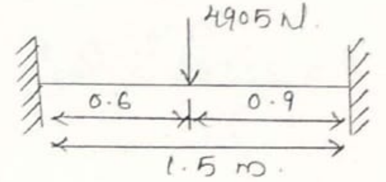
Given datas.

$d = 50\ mm$

$= 0.05\ m$

$m = 500\ kg \Rightarrow W = 4905\ N.$

$E = 200\times10^9\ N/m^2$



$$f_n = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}} \qquad I = \frac{\pi}{64}d^4 = \frac{\pi}{64}\times(0.05)^4 = 3.0679\times10^{-7}\ m^4$$

$$\delta = \frac{Wa^3b^3}{3EIL^3}$$

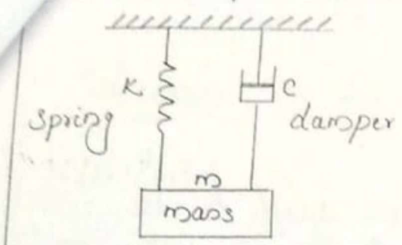
$$= \frac{4905\times0.6^3\times0.9^3}{3\times200\times10^9\times3.0679\times10^{-7}\times1.5^3}$$

$$= 1.2432\times10^{-3}\ m.$$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{9.81}{1.2432\times10^{-3}}}$$

$$= 14.1376\ Hz$$

~ree damped vibration



There are 3 components for a free damped vibration system.

1. mass m, 2. Spring k, 3. damper c - damping coefficient.

There are 3 forces in the system

1. Inertia force $m\ddot{x} = F_i$
2. Damping force $F_d = c\dot{x}$
3. Spring force $F_s = kx$.

For equilibrium,

Total force $= 0$

ie; $F_i + F_d + F_s = 0$

$$m\ddot{x} + c\dot{x} + kx = 0.$$

The above equation is known as equation of motion for free damped vibration.

$$m\ddot{x} + c\dot{x} + kx = 0$$

$$\ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = 0$$

$$\frac{d^2x}{dt^2} + \frac{c}{m}\frac{dx}{dt} + \frac{k}{m}x = 0$$

Put $\frac{d}{dt} = D$ and $\frac{d^2}{dt^2} = D^2$

$$D^2x + \frac{c}{m}Dx + \frac{k}{m}x = 0$$

$$x\left(D^2 + \frac{c}{m}D + \frac{k}{m}\right) = 0$$

$$D^2 + \frac{c}{m}D + \frac{k}{m} = 0$$

Put $D = \alpha$

$$\alpha^2 + \frac{c}{m}\alpha + \frac{k}{m} = 0$$

It's of the form $ax^2 + bx + c = 0$

∴ Soln. is given by $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$\alpha_{1,2} = \frac{-\frac{c}{m} \pm \sqrt{\frac{c^2}{m^2} - 4\left(1 \times \frac{k}{m}\right)}}{2 \times 1}.$$

$$\alpha_{1,2} = -\frac{c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \left(\frac{4k}{4m}\right)}$$

$$= -\frac{c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \left(\frac{k}{m}\right)^2}$$

$$\alpha_1 = -\frac{c}{2m} + \sqrt{\left(\frac{c}{2m}\right)^2 - \left(\frac{k}{m}\right)^2}$$

$$\alpha_2 = -\frac{c}{2m} - \sqrt{\left(\frac{c}{2m}\right)^2 - \left(\frac{k}{m}\right)^2}$$

Since, there are 2 different soln then

$$x = Ae^{\alpha_1 t} + Be^{\alpha_2 t}$$

$$= Ae^{\left(-\frac{c}{2m} + \sqrt{\left(\frac{c}{2m}\right)^2 - \left(\frac{k}{m}\right)^2}\right)t} + Be^{\left(-\frac{c}{2m} - \sqrt{\left(\frac{c}{2m}\right)^2 - \left(\frac{k}{m}\right)^2}\right)t}$$

1. Damping ratio or damping factor (ξ)

$$\xi = \sqrt{\frac{\left(\frac{c}{2m}\right)^2}{\left(\frac{k}{m}\right)}}$$ We have $\omega_n^2 = \frac{k}{m}$

$$= \sqrt{\frac{\left(\frac{c}{2m}\right)^2}{\omega_n^2}}$$

$$\xi = \frac{\frac{c}{2m}}{\omega_n}$$

$$\xi = \frac{c}{2m\omega_n}$$

If $\xi < 1$ System is said to be under damped system

$\xi = 1$ System is said to be Critical damped system

$\xi > 1$ System is said to be Over damped system.

2. Critical damping coefficient (C_c)

It is the value of damping Coefficient c when damping factor $\xi = 1$.

We have, $\xi = \frac{c}{2m\omega_n}$

$$1 = \frac{C_c}{2m\omega_n}$$

$$C_c = 2m\omega_n$$

$\therefore \xi = \frac{c}{c_c}$

Solution in terms of 'ξ'

We have soln,

$$x = A e^{\left[-\frac{c}{2m} + \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}\right]t} + B e^{\left[-\frac{c}{2m} - \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}\right]t}$$

We have, $\xi = \frac{c}{2m\omega_n}$ $\quad \frac{k}{m} = \omega_n^2$

$$\Rightarrow \frac{c}{2m} = \xi\omega_n$$

$$-\frac{c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \left(\frac{k}{m}\right)} =$$

$$-\xi\omega_n \pm \sqrt{\xi^2\omega_n^2 - \omega_n^2}$$

$$= -\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}$$

$\therefore$ solution $x = A e^{\left(-\xi\omega_n + \omega_n\sqrt{\xi^2-1}\right)t} + B e^{\left(-\xi\omega_n - \omega_n\sqrt{\xi^2-1}\right)t}$

for case. 1 $\xi > 1$ over damping

Soln

$$x = A e^{\left[-\xi\omega_n + \omega_n\sqrt{\xi^2-1}\right]t} + B e^{\left[-\xi\omega_n - \omega_n\sqrt{\xi^2-1}\right]t}$$

ie; $x = A e^{\left[-\xi + \sqrt{\xi^2-1}\right]\omega_n t} + B e^{\left[-\xi - \sqrt{\xi^2-1}\right]\omega_n t}$

for case. 2 $\xi = 1$ ~~or~~ Critical damping

$$x = A e^{\left[-\omega_n + \omega_n \times 0\right]t} + B e^{\left[-\omega_n - \omega_n \times 0\right]t}$$

$$= A e^{-\omega_n t} + B e^{-\omega_n t}$$

$$= (A+B) e^{-\omega_n t}$$

for case. 3 $\xi < 1$ Under damping

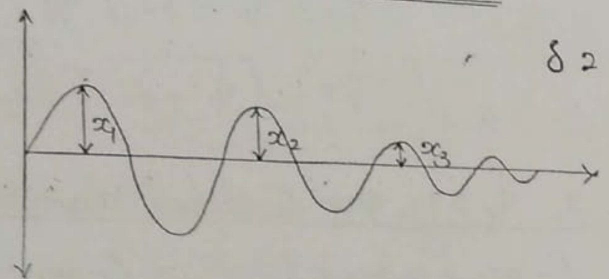
$$x = A e^{\left[-\xi\omega_n + \omega_n\sqrt{\xi^2-1}\right]t} + B e^{\left[-\xi\omega_n - \omega_n\sqrt{\xi^2-1}\right]t}$$

$$= A e^{\left[-\xi\omega_n + \omega_n i\sqrt{1-\xi^2}\right]t} + B e^{\left[-\xi\omega_n - \omega_n i\sqrt{1-\xi^2}\right]t}$$

Put $\omega_d = \sqrt{1-\xi^2}\,\omega_n$

$x = A e^{[-\xi \omega_n + \omega_d i]t} + B e^{[-\xi \omega_n - \omega_d i]t}$

<u>Logrithmic decrement (δ)</u>



$$\delta = \frac{2\pi \xi}{\sqrt{1-\xi^2}}$$

It is the logarithmic ratio of two successive amplitudes.

ie; $\delta = \log\left(\frac{x_1}{x_2}\right)$ or $\log\left(\frac{x_2}{x_3}\right)$

$\delta = \log\left(\frac{x_n}{x_{n+1}}\right)$; generally.

* A vibrating system consists of a mass of 50 kg, A spring with a stiffness of 30 kN/m and a damper. The damping provided is only 20% of critical value. Determine

a) Damping factor
b) Critical damping coefficient
c) Natural frequency of damped vibra
d) Logarithmic decrement.
e) Ratio of 2 Consecutive amplitude.

Given datas.

mass $m = 50$ kg

$K = 30 \times 10^3$ N/m.

$C = 20\%\ C_c \Rightarrow C = 0.2 C_c$

a) Damping factor

$\xi = \frac{C}{C_c} = 0.2$

b) Critical damping coefficient

$\xi = \frac{C}{C_c} =$

$C_c = 2m\omega_n$

$= 2m\sqrt{\frac{K}{m}}$

$= 2 \times 50 \sqrt{\frac{30 \times 10^3}{50}}$

$= 2449.4897$.

$\omega_n^2 = \frac{K}{m}$

$\omega_n = \sqrt{\frac{K}{m}}$

c) Natu vibrat

$f_d =$

c) Natural frequency of damped vibration

$$f_d = \frac{\omega_d}{2\pi}$$

$$f = \frac{\omega}{2\pi}$$

$$\omega = 2\pi f$$

$$\omega_d = \sqrt{1-\xi^2}\,\omega_n$$

$$= \sqrt{1-0.2^2} \times \sqrt{\frac{30\times10^3}{50}}$$

$$= 24 \text{ rad/sec}$$

$$\therefore f_d = \frac{24}{2\pi} = 3.8197 \text{ Hz}$$

d) Logarithmic decrement

$$\delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}} = \frac{2\pi\times0.2}{\sqrt{1-0.2^2}}$$

$$= 1.2825$$

e) $\delta = \cancel{\log}\ \ln\left(\frac{x}{x_{n+1}}\right)$

$$\therefore \frac{x}{x_{n+1}} = e^{\delta}$$

$$\frac{x}{x_{n+1}} = 3.6056$$

Determine a) stiffness of spring
b) Logrithmic decrement
c) damping factor
d) damping coefficient

for a single degree damped vibration system, a suspended mass of 8kg makes 30 oscillation in 18 sec. The amplitude decreases to 0.25 of initial value after 5 oscillations

Given datas

$m = 8kg$

no. of cycles $= 30$; Period $= 18$ sec. $\Big\}\ f = \frac{30}{18} = 1.66\ Hz$

a) stiffness k

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$f_n^2 = \frac{1}{4\pi^2}\frac{k}{m}$$

$$\therefore k = f_n^2 \times 4\pi^2 \times m$$

$$= 1.66^2 \times 4\pi^2 \times 8 = 870.2938\ N/m$$

b) Logrithmic decrement

$$\delta = \ln\left(\frac{x_1}{x_2}\right)$$

$x_6 = 0.25x_1$

$$\frac{x_1}{x_6} = \frac{1}{0.25}$$

$$\frac{x_1}{\cancel{x_2}} \times \frac{\cancel{x_2}}{\cancel{x_3}} \times \frac{\cancel{x_3}}{\cancel{x_4}} \times \frac{\cancel{x_4}}{\cancel{x_5}} \times \frac{\cancel{x_5}}{x_6} = \frac{1}{0.25}$$

$$\frac{x_1}{x_2} \times \frac{x_1}{x_2} \times \frac{x_1}{x_2} \times \frac{x_1}{x_2} \times \frac{x_1}{x_2} = \frac{1}{0.25}$$

$$\left(\frac{x_1}{x_2}\right)^5 = \frac{1}{0.25}$$

$$\frac{x_1}{x_2} = 1.3195$$

$$\therefore \delta = \ln(1.3195)$$

$$= 0.277$$

c) Damping factor

$$\xi = \frac{c}{2m\omega_n} \quad \text{or} \quad \xi\ \cancel{\%}\ \delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$$

$$0.277 = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$$

$$0.277^2 = \frac{4\pi^2\xi^2}{1-\xi^2}$$

$\zeta_1 = 0.044$

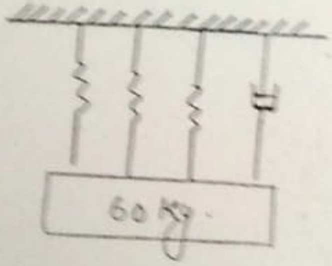
d) $\zeta_1 = \dfrac{c}{2m\omega_n}$

$c = \zeta_1 \times 2m\omega_n$

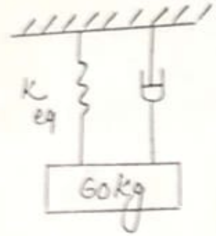
$= 0.044 \times 2 \times 8 \times \sqrt{\dfrac{870.2938}{8}}$

$= 7.3427$

* A machine mounted on spring and fitted with a dash board has a mass of 60 kg. There are 3 springs each of stiffness 12 N/mm, amplitude of vibration reduces from 45 to 8 in 2 oscillations. Determine
  a) Damping coefficient
  b) Ratio of freq. of damped and undamped oscillations
  c) Time period of damped vibration.



$m = 60kg$



$k_{eq} = 3 \times k$

$= 3 \times 12 \times 10^3$

$= 36 \times 10^3 \ N/m$

a) Damping coefficient

$c = 2m\omega_n \zeta_1$

$= 2m\sqrt{\dfrac{k}{m}}\,\zeta_1$

$\omega_n = \sqrt{\dfrac{k}{m}}$

$= \sqrt{\dfrac{36 \times 10^3}{60}}$

$= 24.4948$ rad/s

Given $x_1 = 45$

$x_3 = 8$

$\dfrac{x_1}{x_3} = \dfrac{45}{8}$

$\dfrac{x_1}{x_2} \times \dfrac{x_2}{x_3} = \dfrac{45}{8}$

$\dfrac{x_1}{x_2} \times \dfrac{x_1}{x_2} = \dfrac{45}{8}$

$\left(\dfrac{x_1}{x_2}\right)^2 = \dfrac{45}{8} \Rightarrow \dfrac{x_1}{x_2} = 2.37$

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$\delta = \ln\left(\frac{x_1}{x_2}\right)$

$= \ln(2.37)$

$= 0.8636.$

$\xi_y \Rightarrow \delta = \frac{2\pi \xi_y}{\sqrt{1-\xi_y^2}}$

$0.8636 = \frac{2\pi \xi_y}{\sqrt{1-\xi_y^2}}$

Solving weget $\xi_y = 0.1361$

$\therefore c = 2 \times 60 \times 24.4948 \times 0.1361$

$= 400.049$ ~~Att~~

b) Ratio of freq. of damped and undamped

$\frac{f_d}{f_n} = \frac{\frac{\omega_d}{2\pi}}{\frac{\omega_n}{2\pi}} = \frac{\omega_d}{\omega_n}.$ $\omega_n = 24.4948$ rad/s

$\omega_d = \sqrt{1-\xi_y^2}\ \omega_n$

$= \sqrt{1-0.1361^2} \times 24.4948 = 24.26$

$\frac{f_d}{f_n} = \frac{24.26}{24.4948} = 0.99.$

c) Time period of damped oscillation

$T_d = \frac{2\pi}{\omega_d} = \frac{2\pi}{24.26}$

$= 0.2589$ sec.

☆ A machine weighs 18kg and is supported on spring and dashboard. The total stiffness of the spring is 12 N/mm and damping coefficient is 0.2. The system is initially at rest and a velocity of 120 mm/s, determine

a) displacement and vel ~~of 120 mm/s~~ of mass as function of time.

b) displacement and vel. after 0.4 sec.

Given datas.

$m = 18$ kg $\qquad v = 120 \times 10^{-3}$ m/s.

$K = 12 \times 10^{3}$ N/m

$c = 0.2$

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a) Displacement $x = Ae^{\alpha_1 t} + Be^{\alpha_2 t}$

$$x = Ae^{[-\xi_1\omega_n + i\omega_d]t} + Be^{[-\xi_1\omega_n - i\omega_d]t}$$

$$\omega_n = \sqrt{\frac{K}{m}}$$

$$= \sqrt{\frac{12\times10^3}{18}} = 25.8198 \text{ rad/s}$$

$$\omega_d = \sqrt{1-\xi_1^2}\,\omega_n$$

$$\xi_1 = \frac{C}{2m\omega_n} = \frac{0.2}{2\times18\times25.8198}$$

$$= 2.152\times10^{-4}$$

$$\omega_d = \sqrt{1-(2.152\times10^{-4})^2}\times25.8198$$

$$= 25.8198 \text{ rad/sec}$$

$$x = Ae^{[-2.152\times10^{-4}\times25.8198 + i(25.8198)]t} + Be^{[-2.152\times10^{-4}\times25.8198 - i(25.8198)]t}$$

$$= Ae^{[-5.556\times10^{-3} + 25.8198i]t} + Be^{[-5.556\times10^{-3} - 25.8198i]t} \longrightarrow (1)$$

$$v = \frac{dx}{dt}$$

$$v = Ae^{[-5.556\times10^{-3} + 25.8198i]t} \times [-5.556\times10^{-3} + 25.8198i] + Be^{[-5.556\times10^{-3} - 25.8198i]t} \times [-5.556\times10^{-3} - 25.8198i] \longrightarrow (2)$$

b) when $t = 0$

$x = 0$

$(1) \Rightarrow A + B = 0 \longrightarrow (3)$

$v = 120\times10^{-3}$ m/s ; $t = 0$

$$(2) \Rightarrow 120\times10^{-3} = [-5.556\times10^{-3} + 25.8198i]A + [-5.556\times10^{-3} - 25.8198i]B$$

$A + B = 0 \Rightarrow A = -B$

(2) becomes

$$120\times10^{-3} = [-5.556\times10^{-3} + 25.8198i](-B) + [-5.556\times10^{-3} - 25.8198i]B$$

$$0.12 = \cancel{5.556\times10^{-3}B} - 25.8198iB + \cancel{5.556\times10^{-3}B} - 25.8198iB$$

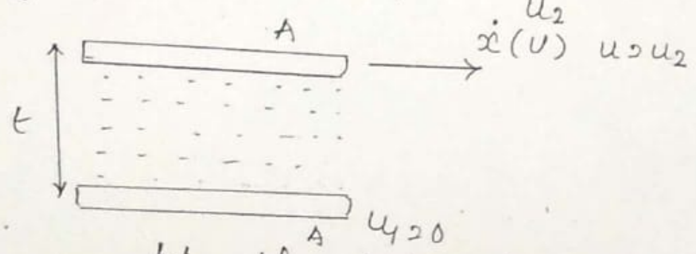
$$0.12 = (-51.6396i)B$$

Types of damping

1. Viscous damping

It is the damping based on the viscosity of fluid

Consider 2 plates of area A seperated by a distance t.



b/w the plates the viscous fluid exists. One plate is fixed and other is moving with a vel $\dot{x}$ or v. We know the Newton's law of viscosity $\tau = \mu \frac{du}{dy}$

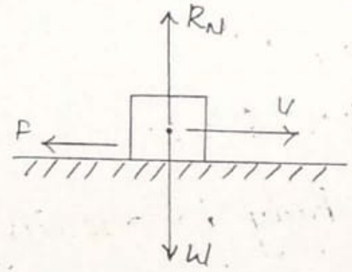
$$\frac{F}{A} = \mu \frac{(u_2 - u_1)}{(y_2 - y_1)}$$

$$\frac{F}{A} = \mu \frac{(u_2 - 0)}{t}$$

$$F = \frac{\mu u A}{t}$$

where F - viscous force
μ - dynamic viscosity
u - vel of upper plate
A - Area of plate
t - distance b/w the plate.

2. Frictional damping (or) Coulomb damping



When an object is moving in a roug plane, the frictional force is develope opposite to the motion. This frictional force is directly proportional to the normal reaction

ie; $F \propto R_N$

$F = \mu R_N$.

μ - Coefficient of friction.

When the frictional force is used for damping, then is called cülomb damping. There are 3 equations of motion in coulomb damping

1. when the body is stationary

$$m\ddot{x} + kx = 0$$

2. when the body is moving towards left

$$m\ddot{x} + kx = F$$

ie; $m\ddot{x} + kx - F = 0$

3. when the body is moving towards right

$$m\ddot{x} + kx = -F$$

ie; $m\ddot{x} + kx + F = 0$